AN INVESTIGATION OF THE USE OF MULTIPLE REPRESENTATIONS IN TEACHING AND LEARNING WHOLE NUMBER MULTIPLICATION IN STANDARD 6

Master of Education: Curriculum and Teaching Studies (Mathematics and Science Education) Thesis

BERTHA MUKWIKWI

Submitted to the department of curriculum and teaching studies. Faculty of Education, in partial fulfillment of the requirements for the degree of master of Education.

UNIVERSITY OF MALAWI

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DECLARATION

I, the undersigned, hereby declare that this thesis is my own work which has not been submitted to any other institution for similar purposes. Where other people's work has been used acknowledgements have been made.

Full Legal Name

Signature

Date

CERTIFICATE OF APPROVAL

The undersigned certify that this thesis represents the student's own work and effort and has been submitted with our approval.

Signature: ________________________ Date: __________________________

Mercy Kazima Kishindo, PhD (Professor)

Main Supervisor

Signature: ________________________ Date: _________________________

Mr. Fraser Gobede

Co Supervisor

DEDICATION

I dedicate this study to my family (my lovely husband, Madalitso Nyambo Kuthakwanasi Chikaonda and daughters, Blessings and Princess), my mother, my sister and brothers.

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Glory and honor be to God almighty for being with me from the time I was born up to now. I sincerely thank my supervisors; Professor Mercy Kazima Kishindo and Mr. Fraser Gobede for the tireless assistance of giving me the necessary direction of my study and their constructive criticisms during the entire work.

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Finally, I owe you my husband, Madalitso Nyambo Kuthakwanasi Chikaonda, a lot. I came to know you at Mzuzu University just as a classmate but now you have become everything to me. Your contribution to my success has been immune. You have been by my side financially, spiritually, socially, naming it, the list is endless. I appreciate the love and care you give me. May the almighty God bless our family.

ABSTRACT

The study investigated the use of multiple representations in the teaching and learning of whole number multiplication in standard six. The study was conducted in two primary schools of Dedza district. Standard six students and mathematics teachers for this class were involved. Three standard six mathematics teachers and a total of 213 students were involved in pre-test and post-test and16 of them were selected for interviews. The study used Ball's mathematical knowledge for teaching (Ball, Thames & Phelps, 2008), in particular specialised content knowledge and Skemp's instrumental and relational forms of understanding mathematics (Skemp, 1976), as the conceptual framework. Data was collected through questionnaires, pre-test, post-test, lesson observation and interviews. Upon analysis of data, the study found out that:(i) teachers perceive that using multiple representations in the teaching and learning of mathematics concepts is very important;(ii) repeated addition, array representation and long multiplication are some of the representations that are commonly used in the teaching and learning of whole number multiplication; (iii) teachers have broad knowledge of the multiple representations which are used to multiply single digit numbers but they have limited knowledge of the multiple representations that are used in multiplying multi-digit numbers. The study has also revealed that multiple representations are used in teaching and learning of whole number multiplication by connecting skills and ideas from one concept to another.

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MKT: Mathematical Knowledge for Teaching

SCK: Specialised Content Knowledge

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CHAPTER ONE

INTRODUCTION

1.1 Chapter Overview

This chapter provides a background of the study, problem statement, objectives, significance of the study, research questions, and definition of key words and structure of the study.

1.2 Background

Primary school learners all over the world are being introduced to various mathematical concepts in their respective classes. Soon (n. d.) describes a concept in two categories which are primary and secondary. The primary concept is being described as the one which has been derived from direct sensory experiences. Some of the examples of these concepts include; number, addition and subtraction. He further says that secondary concepts are those that are derived from other concepts. Some of the examples of secondary concept include; multiplication and division.

According to Soon (n. d), multiplication is a secondary concept which is being formed from the concept of addition. For example, 3 multiplied by 8 is equal to 8 + 8 + 8; that is adding 8 three times or 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3; thus adding 3, eight times. Multiplication is generally denoted by a cross "x", a dot ".", or an asterisk "*". From an example of 3 multiplied by 8 above, it may imply that multiplication of whole numbers is a process which involves repeated addition. Furthermore, multiplication of two numbers is equivalent to as many copies of one of them as the value of the other one. For example; 3 multiplied by 8 may be written using three copies of eights or eight copies of threes. Multiplication has one main property of being commutative. This property indicates that, if one starts with writing the multiplier and seconded by the multiplicand or vice versa, the answer is the same, using the example above; 3 x 8 = 8 x 3. Though learners have different abilities, each learner has to be able to use mathematical concepts in which multiplication is one of the concepts in his or her personal life, whether at work or educational levels (A-lastal & Helai, 2015).These authors continue to say that, all the learners need to have an opportunity to apply the steps involved in mathematics in an accurate and brilliant way for solving the mathematical problem in a creative way.

Typically, Malawian learners learn the concept of multiplication through the memorization of multiplication table. Learners just memorize that 6 multiplied by 7 the answer is 42 without knowing how the 42 is arrived at from the multiplication of those numbers. A learner may not understand multiplication of whole numbers relationally, for example, if a teacher presents 102 x 18 only using the long multiplication algorithm. For the learner to understand the concept of multiplication of whole numbers, he or she need to know different representations that may be used in order to get the answer. The learner needs to know how these representations work and why certain steps are carried out. The teacher should explain using more than one representation like using area model, array, rounding and compensation, double multiplication, partition and other representations. This may help to form a picture in the learner's mind from a variety of representations which have the same idea. In the same way, the learner can learn the concept of multiplication of whole numbers of any quantity of digits. The reason for doing this is to motivate or arouse the interest of the learners on mathematics as a whole and for the learner's understanding. There are a number of issues that affect the teaching and learning process of mathematics and one of the most important issues is the number of representations that are used when teaching and learning the concepts Adu-Gyamfi (1993).

Education system in Malawi follows a structure of 8- 4- 4. That is eight years of primary, four years of secondary and four years of tertiary education. The primary education has three sections; which are infants, junior and senior section. The infant section comprises of standards one and two, the junior section comprises of standards three and four and the senior section comprises of standards five, six, seven and eight.

The concept of multiplication of whole numbers is introduced to primary Malawian learners in standard two. These learners learn the multiplication concept from standard two to standard six. In standard two the learners learn to multiply numbers by 2 and 3 with products not exceeding 99. In standard three learners learn to multiply numbers by 4, 5, 6 and 7 with product not exceeding 999. In standard four learners learn to multiply numbers by 8, 9 and 10 with product not exceeding 9,999. In standard five learners learn to multiply numbers by two digit numbers with the product not exceeding 999,999. In standard six learners learn to multiply four-digit numbers by three-digit numbers with product not exceeding 10,000,000 (MOEST, 2005).

1.3. Statement of problem

Though Mathematics is learnt by all learners in both primary and secondary education in Malawian schools, some learners find it difficult and many do not find it interesting which makes them to take mathematics as a subject that needs to be studied by a special group of people. These learners find mathematics impossibly hard (Devlin, 2000) and many of them openly admit strong dislike for the subject (Paulos, 1988). This may result in a few gifted individuals to have a special inclination towards mathematics subject. Explicitly, a relatively large number of learners do not understand the concepts that are taught due to different reasons. One of the reasons that affect learners understanding is the number of representations that are used by teachers in the teaching and learning process which may help the learners to comprehend what is being learnt and taught. Using one representation does not capture the interest of all the learners. Devlin (2000) claims that, mathematics can be interesting and understood for every learner. When a teacher uses more than one representation, other learners are captured within the lesson. For learners to understand the concept, they should be in a position to link the different representations used in the teaching and learning process. For example as suggested by Salkid (2007), teacher's use of representations is a factor in learners' understanding of mathematical concepts. However, there is insufficient literature on the use of multiple representations in teaching and learning of mathematical concepts in Malawi but extensive literature from other countries on the use of multiple representations can be accessed. It is not known how teachers use multiple representations in Malawi. This study is filing that gap (of limited literature and studies in Malawi) by studying use of representations in Malawi primary education. The copies of this study will be found in hard copies in the library rather than getting the literature on internet as it is for the ones outside the country.

1.4. Purpose of the study

From the statement of the problem above, this study sought to investigate the use of multiple representations in teaching and learning of whole number multiplication in standard 6. To achieve this, the study investigated the perception of teachers on the use of multiple representations of mathematical concepts, knowledge of teachers on multiple representations, different types of representations that are used on multiplication of whole numbers in primary school and how learners understand the concept from the representations.

1.5. Specific objectives

Specifically, this study attempted to:

1. Investigate the experiences of teachers on the use of representations
2. Find out the knowledge of teachers on representation of multiplication concept
3. Find out how learners understand the multiplication concept from the representations.

1.6. Research questions

This study was designed to answer the question; how are multiple representations used in teaching and learning of whole number multiplication? The following sub questions were set to answer the main question;

1. What are the perceptions of teachers on the use of multiple representations?
2. What representations of multiplication do teachers use?

3. What knowledge of multiple representations do teachers have?
4. How do teachers select representations of multiplication?
5. How do learners understand the whole number multiplication concept from the multiple representations?

1.7. Significance of study

The findings of this study may inform different stakeholders as presented below;

1.7.1. The researcher and other mathematics teachers

It is believed that the researcher and other mathematics teachers would be informed about the significance of using multiple representations in the teaching and learning of mathematics. This may make teachers to search for different representations of a given concept when planning for their work in order to capture the interest of the learners.

1.7.2. Mathematics Teacher Educators

The findings of this study may inform mathematics teacher educators to train prospective mathematics teachers on the use of multiple representations on teaching and learning mathematical concepts.

1.8. Definition of terms

Representations: Means by which individuals make sense of situations" (Kaput, 1989, p. 46). Kaput added that representations may be a combination of something written on paper, something existing in the form of physical objects, or a carefully constructed arrangement of ideas in one's mind.

Multiple Mathematical representations: **"**Mathematical embodiments of ideas and concepts that provide the same information in more than one form" (Ozgun-Koca, 1998, p 3).

Understanding: Being able to recognise the concept in a variety of different representations, manipulate the concept within given representations and translate the concept from one representation to another (Janvier, 1987).

1.9. Structure of the thesis

This thesis reports the study that has investigated the use of multiple representations in teaching and learning whole number multiplication in standard 6. The thesis contains five chapters: Introduction, literature review, research design and methodology, findings and discussion and conclusion.

The introduction has described the background to the topic of the study. The chapter has also described the problem statement, purpose of the study, specific objectives, research questions and the significance of the study. The chapter has also considered the definition of key terms as used in the study. The chapter on literature review has presented issues related to the use of multiple representations like types of representations, factors to be considered when selecting a representation to be used and understanding. The chapter has also discussed the conceptual frame work of the study. The chapter of research design and methodology has discussed the research design, study area, study population, study sample size, sampling techniques, pilot study, data collecting instruments, data analysis and presentation, ethical consideration and limitations of the study. In the findings and discussion chapter, the findings have been discussed according to the research questions.

The conclusion chapter contains summery of findings, implication and recommendations and areas for further study.

1.10. Chapter Summary

This chapter has described the background to the topic of the study. The chapter explained what the study investigated by stressing the problem in the teaching and learning of mathematics and the importance of using multiple representations. Five research questions that are guiding the study, definition of key words and structure of the study have been presented.

CHAPTER TWO

LITERATURE REVIEW

2.1. Introduction

The review of this study has covered six parts. The first part is on definitions of representations and multiple mathematical representations. Secondly, the types of multiple representations which are; enactive, icon and symbolic (Bruner, 1966) have been discussed. In addition to this, the review has covered the teacher's knowledge of mathematical representations. Further to this, the review has also discussed the factors that are considered when selecting mathematical representation of a concept. The factors include; stage of development of learners and the learners' prior knowledge on the representation of mathematical concepts (Bruner, 1966; Leinhardt, 1989). Another part of the review has unveiled the meaning of understanding and what a teacher can do to find out if the learners have understood the concept or not. Furthermore, it has discussed several studies that have been carried out by different researchers on the use of multiple representations.

2.2. Representations

According to (Kaput, 1989), representations are means by which individuals make sense of situations. These representations may be a combination of something written on paper, something existing in the form of physical objects, or a carefully constructed arrangement of ideas in ones' mind. Learners develop representations in order to interpret and remember their experiences in an effort to understand the world. In mathematics teaching, representations are helpful tools that support mathematical reasoning, facilitate mathematical communication, and convey mathematical thought (Kilpatrick, Swafford, &Findell, 2001; Zazks, 2005). When teachers use mathematical representations during the teaching and learning process, learners use these representations to support understanding when they are solving mathematical problems or learning new mathematical concepts. In addition, the use of representations such as objects, pictures, symbols, and gestures has been found to be helpful in clearing up learners' mathematical confusions (Flevares & Perry, 2001).

2.3. Multiple mathematical representations

According to Ozgun-Koca (1998, p.3), "Multiple mathematical representations are mathematical embodiments of ideas and concepts that provide the same information in more than one form." In other words, using multiple representations simply means using more than one representation. As far back as the early 1920's, the National Committee on Mathematical Requirements of the Mathematics Association of America in their reorganization of mathematics report of 1923, recommended that learners develop the ability to understand and use different representations to solve algebraic and geometric problems (Bidwell & Clason, 1970). During the early 1970's, Dienes (1971) suggested that mathematical concepts need to be presented in as many different forms as possible in order for learners to obtain the mathematical essence of an abstraction which he calls multiple embodiment principle. Dienes (1971) contended that using a variety of representations to develop mathematical concepts maximizes learners' learning.

2.4. General types of representations

According to Bruner (1966), there are three distinct ways in which people represent the world. The ways include; through action, through visual images and through words and language. He called these kinds of representations enactive, iconic, and symbolic, respectively. Most researchers agree that these three types of representations are important in human understanding. Other researchers have reduced the three types to two categories (Clark & Paivio, 1991; Marzano, 2004; Marzano, Pickering, & Pollock, 2001) which are internal and external while others have included additional categories (Lesh, Landau, & Hamilton, 1983). Dual coding theory maintains that there are two systems of representation (verbal and visual) that allow the brain to process and store information in memory (Clark & Paivio, 1991). The interconnectivity of the verbal and visual coding systems allows information retrieval to occur easily. These two systems have also been called linguistic and non-linguistic (Marzano, 2004; Marzano, Pickering, & Pollock, 2001). Lesh, Landau, and Hamilton (1983) contribute that there are five kinds of representations which are; real life experiences, manipulative models, pictures or diagrams, spoken words, and written symbols.

2.5. Types of mathematical representations

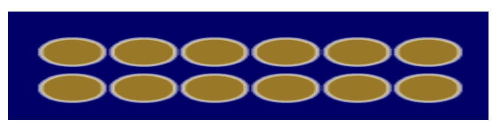
"Mathematics requires representations for the concepts to be understood. Based on the abstract nature of mathematics, most people have an access to mathematical ideas only through the representation of those ideas" (Kilpatrick, Swafford, & Findell, 2001, p. 94). Representations of mathematical concept include objects, actions, pictures, symbols, and words. These representations could be linked to Bruner's three types of representations, with objects and actions being enactive, pictures being iconic, and symbols and words being symbolic.

2.5.1. Enactive mathematical representation

This is the type of representation in which objects are used in form of action (Bruner, 1966). For example, to teach 2 x 6, a teacher may involve learners to use objects and arrange them in two rows by six columns or in six rows by two columns and count the number of objects that have been involved. The following are some of the multiple representations for 2 x 6 under enactive.

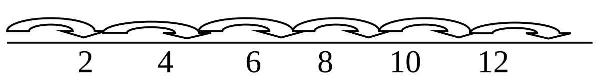
1. **Array representations**

(a) **(b)**



In these representations, learners arrange the objects according to the factors given. In this case the objects are arranged in 6 columns and 2 rows. After counting, the learners may be able to find that there are 12 objects that were involved.

2. **Number line or skip counting**



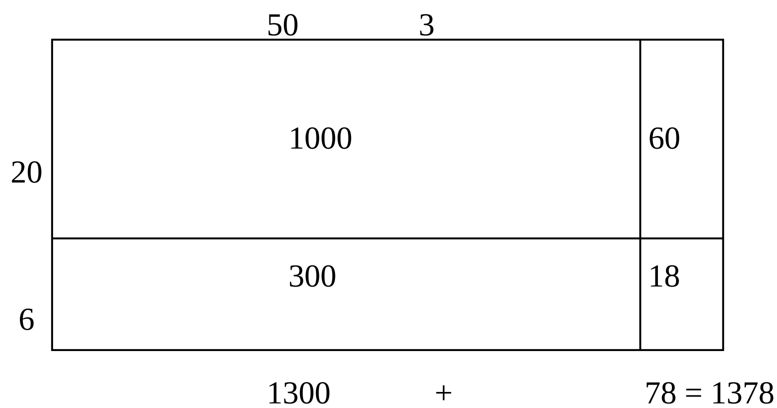
In this representation learners use 6 steps of 2s which gives 12 or learners are asked to count in 2s from 0 and count 6 times. In this case the counting goes; 2, 4, 6, 8 10, 12.

Since at 12 it is where we are having the sixth count, the counting stops there, and that gives the answer (Wallace & Gurganus, 2005).

2.5.2. Iconic Representations

This is a type of representation in which visual images or pictures are used (Bruner, 1966). For example, a teacher may use an area model, grid method or crossed lines when multiplying the given figures. The following are some of the multiple representations of 53 x 26 under iconic representation.

a. Partitioned rectangular model



In this representation, the factors which are 53 and 26 are partitioned to 50 and 3 and 20 and 6 respectively. The partitioned factors are written by the sides of the partitioned rectangle as it is shown above. The rectangle has been partitioned into 4 rectangles of sides; 50 by 20, 50 by 6, 20 by 3 and 6 by 3 if we are to start with the lengths of the four rectangles. The areas of the four rectangles which are also referred to as partial products (Bruner, 1966) of the whole rectangle are calculated as1000, 300, 60 and 18 respectively. After adding, the result of 1378 is found. The pictures that are used in the representations help learners to learn better, (Bostrom & Lassen, 2006).

b. Grid partitioning

	50	3	
20	1000	60	1060
6	300	18	+ 318
			1378

In grid partitioning, the factors are also partitioned and these partitioned factors are put in their own grid (Salkind, 2007). The partial products are found by multiplying 50 by 20, 50 by 6, 20 by 3 and 6 by 3.

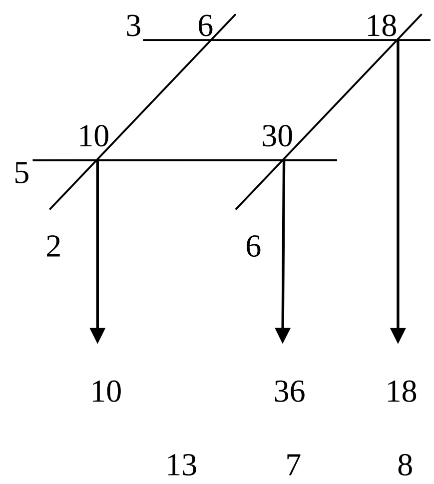
The products as above are found and added together giving 1378. This representation can also be done as below;

	50	3	
20	1000	60	
6	300	18	

1300 + 78 = 1378

The difference of this representation from the one above is that in the first one, two partial products are added across the rows e.g. 1000 + 60 and 300 + 18 before being added together while the latter, two partial products are added along the columns e.g. 1000 + 300 and 60 + 18 before the final addition.

c. **Place value representation**



In this representation, numbers in the factors are written according to their place values in slanting vertical and horizontal lines (Wallace & Gurganus, 2005). In this case, for 53 there are 5 tens and 3 ones while 26 is 2 tens and 6 ones. 2 tens multiplied by 5 tens the result is 10 hundreds and the ten is written on the diagonal which is closer to the 5 and 2. 2 tens multiplied by 3 one gives 6 tens and is written on the diagonal closer to 3. 6 ones multiplied by 5 tens gives 30 tens and it is written on the diagonal near 6 and 6 ones multiplied by 3 ones we get 18 ones which is written on diagonal of 3 and 6. Numbers on the same position of diagonals are added together, in this case 18 is alone on the ones diagonal so it is dropped. 6 and 30 are on the tens diagonal which are adding up to 36 tens. 10 is also alone on the diagonal of hundreds and it is dropped now giving 10 hundreds, 36 tens and 18 ones. In 18 there is 1 ten and 8 ones so the 8 is written on the ones. The 1 ten is carried on to the 36 tens adding up to 37 tens. In 37 tens there are 3 hundreds and 7 tens so the 7 is written on the tens. The 3 hundreds are carried on to the 10 hundreds adding up to 13 hundreds and 13 is written. At the end we get the answer 1378 as found in the other representations.

2.5.3. Symbolic representations

This is the type of representation in which words and symbols are used (Bruner, 1966). In this type of representation, a teacher may use long multiplication, partitioning, rounding and compensating doubling and words. The following are the representations for 78 x 19 in symbolic representations.

a. Partitioning

$$
\begin{array}{r}
78 \times 19 = 70 + 8 \\
\underline{\times\ 10 + 9} \\
700 \\
80 \\
630 \\
\underline{+\ 72} \\
1482
\end{array}
$$

In partitioning, the two factors are partitioned to 70 + 8 and 10 + 9 respectively. 10 multiplied by 70 gives 700, 10 multiplied by 8 gives 80, 9 multiplied by 70 give 630 and 9 multiplied by 8 gives 72. Adding 700 to 80 to 630 to 72 gives 1482

b. Long multiplication

$$
\begin{array}{r}
78 \\
\underline{\times\ 19} \\
702 \\
\underline{+78} \\
1482
\end{array}
$$

This representation is also known as common algorithm (Barmby, Bolden, Raine & Thompson, 2013). It is common to everyone who has learnt multiplication of whole numbers. For this representation 9 ones are firstly multiplied by 8 ones which give 72 ones and the 7 tens are carried on writing the 2 ones. The 9 ones multiplied by 7 tens the result is 63 tens and adding to the 7 tens that were carried on gives 70 tens. 1 ten multiplied by

8 ones the result is 8 tens so the 8 is written below 0 which is the tens value. 1 ten multiplied by 7 tens gives 7 hundreds and the 7 is written under 7 which is the hundreds value. Adding 7 hundreds and 2 ones to 7 hundreds and 8 tens the result is 1482 which is one thousand, four hundreds, 8 tens and two ones.

c. Rounding and compensating

78 x 19 = 80 x 20 - (2 x 20 + 1 x 78)

= 1600 - (40 + 78)

= 1600 - 118

= 1482

In this representation, both factors are rounded up to the nearest place value (Wallace & Gurganus, 2005) in this case the tens. 78 is rounded up to 80 and 19 is rounded up to 20. Products of how much has been added to 78 to make 80 multiplied by 20 and how much is being added to 19 to make 20 multiplied by 78 are found and their sum is subtracted from the product of 80 and 20. 80 multiplied by 20 give 1600. 2 multiplied by 20 give 40 and 1 multiplied by 78 gives 78. Adding 40 to 78 gives 118 and subtracting 118 from 1600 the result is 1482. This can also be done by first finding the product of 80 and 20 then subtract the sum of the products of 2 and 19 and 1 and 80. 80 multiplied by 20 gives 1600, 2 multiplied by 19 give 38 and 1 multiplied by 80 gives 80. Adding 38 to 80 gives 118 which gives 1482 when subtracted from 1600 as below;

78 x 19 = 80 x 20 - (2 x 19 + 1 x 80)

= 1600 - (38 + 80)

= 1600 - 118

= 1482

d. Double multiplication

78 x19

1	19
*2	38
*4	76
*8	152
16	304
32	608
*64	1216

We start from one on the left column and double continuously up to when we reach at a number that if doubled, it may exceed the multiplier which is 78. On the right column, we start from the multiplicand (19 in this case) and double it continuously up to where we have stopped on the left. Adding the products of 2 and 19, 4 and 19, 8 and 19 and 64 and 19 in which 2, 4, 8 and 64 add up to 78, the sum of the products which are 38, 76, 152 and 1216 is 1482. Since multiplication has a commutative property, 79 x19 is equal to 19 x 78. The double multiplication for 19 x 78 is represented as follows;

19 x 78

*1	78
*2	156
4	312
8	624
*16	1248

8 624

Adding the products of 1 and 78, 2 and 78, and 16 and 78 in which 1, 2 and 16 add up to 19, the sum of the products which are 78, 156 and 1248 is 1482.

e. Word representation

78 multiplied by 19 can also be represented as Seventy eight nineteens or nineteen seventy eights.

Goldin & Shteingold (2001) have two systems of representations; external and internal. The external systems of representation include conventional representations that are usually symbolic in nature. The internal systems of representation are created within a person's mind and used to assign mathematical meaning. Our numeration system, mathematical equations, algebraic expressions, graphs, geometric figures, and number lines are examples of external representations. These representations have been developed over time and are widely used. External representations also include written and spoken language. Examples of internal representations include personal notation systems, natural language, visual imagery, and problem solving strategies. This study will focus on the external representation which encompasses enactive, iconic and symbolic representations.

2.6. Teachers' Knowledge of Representations

Salkind (2007) argues that teachers who are effective know how mathematical ideas can be represented in order to facilitate learners' understandings of those ideas. Shulman (1986) suggests pedagogical content knowledge as a specialized domain of content knowledge that teachers need for teaching. Together with knowledge of the topics of instruction within one's subject area, an understanding of what makes those ideas simple or hard to grasp, and learners' common misconceptions within those topics; pedagogical content knowledge includes "the most useful forms of representation of those ideas, the most influential analogies, illustrations, examples, explanations, and demonstrations using words, the ways of representing and formulating the subject that make it understandable to others" (Shulman, 1986, p. 9). Shulman sees representations as an important part of pedagogical content knowledge.

Mathematical Knowledge for Teaching is a deep understanding of mathematics that allows teachers to explain why common algorithms work. An important part of mathematical knowledge of teaching is the ability to generate and use representations (Ball, Thames & Phelps, 2008). Teachers need to be able to translate complex mathematical ideas into representations that learners can understand (Fennema & Franke, 1992; Orton, 1988). In order to do so, teachers need to have a collection of representations that are useful for teaching mathematics which would include story problems, pictures, situations, and concrete materials (Ball, 1990). Teachers also need to understand the strengths and weaknesses of different representations and how they are related to one another (NCTM, 2000).

2.7. What to Consider When Selecting Representation

2.7.1. Stage of development

According to Bruner (1966), different stages of human development stress different representational systems. Young children learn through manipulation and action also known as enactive representation, older children learn through perceptual organization and imagery also called iconic representation, and adolescents learn through the use of language and symbolic thought know as symbolic representation. This idea has become a staple of school mathematics instruction with teachers knowing that learners must begin with concrete experiences (enactive), move to pictorial representations (iconic), and finally progress to abstract understanding (symbolic). However, Clements (1999) and Wittmann (2013) suggest that all three types of representation should be used in parallel to facilitate learner's learning. When learners make connections amongst concrete, pictorial, and symbolic representations, their learning is enhanced and improved. To construct a more in-depth conceptual knowledge of a concept, lessons need to include all three types of representation (Sunyono, Yuanita, & Ibrahim, 2015). Since Bruner's study was in the 60's his claim was concurring with the ages of learners who were attending primary school in those years. For example in Malawi one could find a primary learner of 17 years of age in standard 2. On the other hand Clements' study was done in the late 90's which is in agreement with the study that was done in 2015 by Sunyono and colleagues. According to age of learners who are in primary schools, for example, nowadays some children of 10 years of age are pursuing primary education in standard 6 so it may be better if the learners are exposed to all the types of representations. This may help the learners of particular level of development to grasp the mathematical concept according to their level.

2.7.2. Student's prior knowledge on representations

In her research, Leinhardt (1989) discovered that experienced teachers use representations that learners already know to teach new content, while novice teachers introduce new representations alongside new content. She also found that expert teachers tend to use the same representations to teach multiple content topics. In addition, novice teachers often struggle to explain topics using representations because they are not familiar with the representations (Debrent, 2013). Implications of the study suggested that it is important for learners to understand the representations that teachers use, familiar representations can be useful for teaching new content, and one representation may be valuable for teaching multiple content topics. Furthermore, as previously discussed above, teachers must be well versed in the representations they use to illustrate mathematical ideas.

2.8. Understanding

Since the term, understanding, has a lot of different meanings, the notion of Janvier, (1987) suggest that a learner who understands a concept is the one who can recognize the concept in a variety of different representations and also can flexibly manipulate the concept within given representations and can translate the concept from one representation to another.

Skemp (1976) distinguished between two kinds of understanding: instrumental and relational. Instrumental understanding is considered as rules without meanings, while relational understanding requires conceptual connections and explaining why the rules work. Skemp discussed certain advantages of encouraging one of instrumental and relational understanding over the other. Instrumental understanding can be beneficial for a short term case within a limited context, whereas relational understanding is better for long term learning in a broader context.

Lamon (2001) in her study of finding the distinction between models of representations that learners use to show their mathematical thinking, she suggests that teachers can evaluate whether or not learners have understood the mathematical concepts by examining the representational models that learners choose to use. She continues that if the learner's representation is different from the one the teacher used, then it can be assumed that the learner understands the concept. Learners that use exact the same representations of the teacher, may be parroting the teacher without real understanding (Piez &Voxman 1997).

From these three authors Janvier (1987), Skemp (1979) and Lamon (2001), it can be deduced that mathematics understanding starts from being able to explain how and why a formula or steps of solving a problem works for a given concept, selecting an appropriate representation for a given problem of a concept and making logical connections among different representations of a concept or a problem.

Several studies have been carried out in different countries on the use of multiple representations in the classroom. One of the studies is the one done in Iran by Sisakht and Larki (n.d.) on the role of using multiple representations in fractions with instructions. This study also investigated the effects of using the multiple representations on Grade 4 learners' understanding. The study was conducted using 40 girl learners in 4^{th} grade who were studying at two elementary schools in Sisakht town, in order to investigate the effects of a multi-representational instruction on the understanding of learners from fraction concepts. It was an experimental design in which the learners of one school randomly were selected as experimental group and the learners of another school selected as control group. In experimental group, the learners learned the concepts of fraction by using multiple representations, whereas the learners in control group learned the same concepts with a traditional approach (the approach that did not emphasise multiple representations). The results of the study indicates that using multiple representations with the process of teaching and learning of mathematical complex concepts such as fractions enhances the relational understanding of the concepts. The authors suggest that learners can improve their ability to compute and conceptualise fractions if fractions instructions emphasize understanding and the use of reform based or the contemporary practices such as applying an instructional approach based on multiple representations.

In New Zealand, Loveridge and Mills (2008) carried out a study on representation in multi digit multiplication using array based materials. The study was done by involving 7 teachers from four elementary schools and some learners where the teachers were visited twice and the learners were given a written assessment on multiplication at the first visit before the lesson. Then teachers were taught on how they can represent multi digit numbers to their students. At the end the learners were interviewed. The study revealed that arrays can be useful for enhancing learners' understanding of multi digit multiplication. It was found that teacher's use of dotty arrays to represent multi digit multiplication as a rectangle with sides corresponding to the two factors is associated with improved understanding on multiplication. These findings are similar to (Davis, 2008, p. 88) who says, "the most flexible and robust interpretation of multiplication is based on dotty array rectangle." An advantage to dotty arrays is that they help learners to appreciate differences in the magnitude of partial products and the impact of place value on the size of sections within an array.

Another study on the impact of using representations on acquisition of mathematical concepts among 6th graders was done by A-lAstal and Helai (2015) in Gaza strip. This study was conducted using an Experimental Design where pre-test and post-test were given to two groups amounting to 80 learners. The two groups were randomly selected from Mustafa Hafez Elementary School "B" which is located in Khan Yunis Governorate. One of them was assigned as an experimental group, and the other as a control group. The study revealed that the use of mathematical representations gives positive results in acquiring mathematical concepts. Depending on these representations learners can integrate between mathematics and real life situations, which makes learning mathematics meaningful and overcome concept misunderstanding in elementary schools.

In North Carolina, a study on external multiple representations in mathematics teaching was done by Adu-Gyamfi (1993). It was done by carrying out an extensive review of the available literature. Findings derived from the review of studies suggest that, incorporating the use of multiple representations in mathematics instructions facilitate learners in their understanding of mathematical relations and concepts. It also helps the learners on how they create their understanding of mathematical relations and the concepts.

In Israel, a study on designing representations: reasoning about functions of two variables was done by Yerushalmy (1997). This study was conducted using an Experimental Design at an urban secondary public school. All the seventh grade algebra learners participated for the first year. The class consisted of 38 learners of a range of ability levels. Software that supports a guided inquiry approach which supplies multiple parallel representations was used. The participants spent several months learning to use the language of processes and events to describe, qualitatively, quantitative situations for functions of a single variable. Afterwards they moved on to describing patterns of numbers using graphs and symbolic notation. They carried out their inquiry by doing written activities, working in pairs and small groups, and participating in whole class discussions. Seven learners were chosen from those who volunteered to participate. These learners were chosen on the basis of their cooperativeness and willingness to discuss their ideas, as demonstrated in their previous class work. Six of them stayed through all stages of the experiment. The mathematics teacher and the researcher told the learners that they were looking for an opportunity to study how they would solve a problem that would not be studied by the whole class and that required creativity. The experiment was structured in four stages: firstly, learners were asked to design representation of a dependency in two variables; secondly, explanations were given by members of the groups; thirdly, learners solved problems while considering the various strategies; and lastly, the researcher and the participants discussed the contributions of various methods to the solutions. The study made clear that the language and representations of functions are not just usable or handy for learners, but are also used naturally to create new mathematics. The study helped the participants to use representations to present a learning episode in which designing and inverting representations turn out to be a natural part of mathematics curriculum within the framework of traditional school content. From this study it can be taken that some of the representations which are discovered from different studies may be useful in the mathematics curriculum for years to come. There is a need to give attention to the different representations that are used by learners in solving some mathematical problems so that these representations may be used in future.

Another study was done in Belgium by Verschaffel (1994) on using retelling data to study elementary school children's representations and solutions of compare problems. It was an experimental design which involved 40 learners of age range 10 to 11 years from two Flemish fifth grade classes. One class consisted of 18 learners and the other of 22 learners.

It was done by administering nine addition and subtraction word problems to each learner. These problems required one step only for them to be solved. The problems consisted of one warming up problem, four compare problems that played the role of target items, and four filler problems that were included to avoid stereotyped responses. The nine problems were printed on cards. The back side of each card contained the two given numbers. Each learner was tested individually and the learners were asked to read each problem silently and then to solve it. There was no time limit since they could read the problem on the card at their own pace and reread it as many times as they wanted before and during the solution process. After answering the problem, the learner was shown the two given numbers on the other side of the card and was asked to retell the problem. This study found out that the accurately retelling of the problem reflects the representation lies at the heart of the learner's choice of operation. The researcher concluded that the learners' use of the appropriate arithmetic operation is based on the representation which is easier for them to solve a given problem. This entails that learners should be exposed to a variety of representations. Different representations may provide a chance to learners to select the representation which is easier to them when solving a mathematical problem.

In Midwestern U.S, a study on children's representations and organization of data was conducted by Nisbet, Jones, Thornton, Langrall and Mooney (2003). The study was done involving 15 learners in grade 1 through 5 in which three were selected from each of the 5 grades. At each grade level, children were purposefully sampled based on their previous mathematics achievement; one high, one middle and one low in order to increase the representativeness of the sample. The first author interviewed all the children in the sample using two researcher- designed statistical representation protocol. For each child, protocol 1 was administered in the first session, and protocol 2 in the second session. Protocol 1 involved data on how a class of 10 learners in a rural school travelled to school. And the participants were asked to draw a picture or graph and the follow-up questions were asked. Protocol 2 involved the number of pet fish belonging to a group of 10learners. The fish data were numerical with each learner listed by name and number of fish and they were asked to draw a picture or a graph to represent the data. A set of follow-up questions were asked based on the drawing. The study revealed that learners in grade 1 were more idiosyncratic and incomplete in their thinking with respect to organizing and representing data than their counterparts in grade 2 to 5. The result points to the importance of mode of presentation and context in data exploration especially with young children. The ability to make connections between different aspects of data enabled learners beyond grade one produce more normative organisations and representations of the data. Learners in grade 2 through 5 were able to use pictographs, bar graphs and tally graphs when representing data. From this study it may be taken that any kind of representation may be used in the teaching and learning process of mathematics concept regardless of the study level of the learners. Furthermore, teachers need to take it that when learners have used a representation, there must be a follow up to see if the learner understands the representation.

In South Africa, a study on a modeling and models approach: improving primary mathematics learner performance on Multiplication was done by (Dlamini, Venkat & Askew,2015). The study involved 33 grade 6 learners. At the beginning of the study, a pre-test which consisted of ten problems was administered to learners. Six of these problems were represented as multiplicative situations; the remaining four items were ‘buffer’ items involving other operations and/or ‘straight’ multiplication calculations in order to avoid ‘cueing’ learners into multiplication calculation strategies. The pre-test was followed by a six week intervention lessons. The intervention focused on repeated addition, multiplication as a rate and multiplication as scaling. After the intervention lessons, a post-test was administered. The results from the learners’ work indicated important shifts in the use of models between the pre- and post-tests. First indications of a limited number of models which were observed in a pre- test results were not observed when the use of a broader range of models within problem-solving were engaged. It was concluded that a teaching approach based on the use of models can have a positive impact on performance and processes in solving word problems in multiplication. However, it can be argued that, use of models may work only if a teacher is able to use the models properly for learners' understanding. If the teacher has no knowledge on how he or she can use the models, then the intended use of the models may not work.

From these studies, there is a clear indication that all learners can learn and understand mathematics if teachers incorporate multiple representations during the teaching and learning process. Every learner's interest can be captured in the process of teaching and learning because one learner may prefer to use one representation and another learner may prefer to use another representation based on the learner’s choice from the representations exposed to them. Though many studies on the use of multiple representations in the teaching and learning of multiplication of whole numbers have been conducted, most of them have based their findings on the performance of the learners on the concepts that have been taught and learnt. Performance of learners may not make a teacher to conclude that the learners have understood a concept because one may get a correct answer to a particular problem just based on memorization. There is a need to investigate more on the reasoning of the learners when they use a particular representation to solve a mathematical problem. Hence this study is interested to investigate the use of multiple representations in the teaching and learning of whole number multiplication, focusing much on how learners justify the steps they have carried out when solving a given problem which involves multiplication of whole numbers.

2.9. Conceptual framework

This study was guided by two sub domains of Ball's mathematical knowledge for teaching, and Skemp's forms of understanding. According to Ball etal. (2008), Mathematical Knowledge for Teaching (MKT) is knowing mathematics from the perspective of helping others to learn it and includes being mathematically ready to teach an idea, method or other aspect. In the domain of subject matter knowledge, Common Content Knowledge (CCK) and Specialised Content Knowledge (SCK) are some of the important sub domains of MKT explained by (Ball etal., 2008). CCK is defined as the mathematical knowledge that is known in common with people who know and use mathematics in different fields. This knowledge makes teachers to know the matter they teach; they are made to detect when their learners give incorrect answers or when the textbook gives an incorrect definition or explanation. Teachers with CCK may be able to use terms and notations correctly. In brief, these teachers may be able to do the work they assign their learners (Ball etal., 2008). This study used these as the indicators of CCK in teachers.

Ball etal. (2008) says that teaching mathematics is a special kind of mathematical work that includes solving special kinds of mathematics problems, engaged in specialised mathematical reasoning, and use of mathematical language in careful ways. This special knowledge of mathematics, which is particular for teaching, is known as Specialised Content Knowledge (SCK). This knowledge makes the teacher to be able to talk explicitly about how to choose, make, and use mathematical representations effectively and how to explain and justify one's mathematical ideas. The teachers who possess this knowledge may be able to present mathematical ideas, answer the why questions of learners, come up with an illustration to compose a particular mathematical idea, recognise the work that is involved in using a particular representation, connect representations to mathematical ideas and to other representations, connect a topic that is being taught to the topics from previous or upcoming years, evaluate and familiarise oneself with the mathematical content of textbooks, restructuring mathematical problems to be either easier or harder, evaluate how plausible learners' answers are, present or evaluate mathematical explanations, select and develop definitions that can be used, use mathematical notation and language and critiquing its use, ask mathematical questions which are authentic and select representations of a mathematical concept for particular reason (Ball, 2011). Since the teachers who possess the SCK have adequate knowledge on the use of multiple representations, this study investigated if teachers have the knowledge of using multiple representations on multiplication of whole numbers. The study used these as indicators to describe the teachers with this knowledge as the teachers with the SCK.

As suggested by Skemp (1976), there are two forms of understanding which are; relational and instrumental understanding. He continues by claiming that relational understanding is to know both what to do in order to solve a mathematical problem and why it is done in that way. On the other hand, he describes instrumental understanding as knowing rules of solving mathematical problems without knowing why the rules work. Furthermore, a learner who has understood the mathematical concept is the learner who may be able to know how a representation works and why. For this reason, the learner who will be able to explain the used representation of multiplication of whole numbers given to him or her has understood the multiplication concept. Hence, the study also managed to find out if learners understand multiplication of whole numbers relationally. According to Skemp (1976), some of the properties of relational understanding include; it is adaptable to new concepts. This is achieved in learners by not only knowing what representation works for a particular concept but also knowing why it works the way it does (Skemp, 1978) and this enables them to relate to new concepts. Another property is that it makes learners to understand the connections between concepts. This happens when ideas that are vital for understanding a given topic or concept turns out to be essential for the understanding of various topics or concepts as well. Another property is that relational understanding helps learners to actively look for new materials and come up with new ideas to suit a given situation. These properties were used as indicators of relational understanding in the study.

Since teachers with (SCK) are able to use mathematical representations effectively and know how to explain and justify one's mathematical ideas, he or she may be able to teach a mathematical concept relationally.

2.10. Chapter Summary

This chapter has shown that there are different representations that may be used in teaching and learning whole number multiplication. The representations include; array, number line, rectangular model, place value, rounding and compensating, double multiplication and long multiplication. The chapter has also outlined factors that are considered when choosing a representation to be used in a particular problem. The chapter has also discussed the usefulness of multiple representations in teaching and learning mathematical concepts. Furthermore, the chapter has considered the connection that exists between the uses of multiple representations and the SCK and relational understanding.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.1. Introduction

This chapter gives an insight of how the study was carried out. The chapter has covered study design, study area, study population, study sample size, sampling techniques, pilot study, data collection instruments, data analysis and presentation, ethical consideration and limitations of the study.

3.2. Study Design

This research was in a form of an experiment which used both qualitative and quantitative designs. Pre experimental is the type of experiment that was used in the study. This type of experiment uses one group which is observed before and after the treatment. The change that is noticed in the observed group is attributed to the treatment used in the experiment (Box, Hunter & Hunter, 2005). The treatment that was used was a workshop on multiple representations which was done by the researcher on the teachers. Teachers re-taught the multiplication concept to their learners using two representations of their choice. According to Creswell (2009), qualitative research is the type of research which involves looking in-depth of a phenomenon at non-numerical data. This type of research helps the researcher to look inside of the respondents minds. On the other hand quantitative research is the systematic empirical investigation of observable phenomenon via statistical, mathematical or computational techniques (Creswell, 2009). Senior section was chosen because of the unavailability of multiple representations on multiplication of whole numbers in text books and the teacher's guides for the senior classes.

3.3. Study Area

The study was conducted in Dedza which is one of the districts in the Central West Education Division. The schools that were involved were taken from Boma zone of the district. This zone was chosen for convenience because it is within the area where the researcher stays.

3.4. Study Population

The population of this research contained both standard six teachers and standard six learners. Since teachers are the ones who teach different concepts to their learners, and learners are expected to understand, it was therefore, important to solicit teachers' views on the use of multiple representations and finding their knowledge on mathematical representations. Learners were involved to check their understanding since it was a hub of this study.

3.5. Study Sample Size

The study involved two primary schools in which 213 standard six learners were involved for the pre-test and the post test. Standard six was chosen in order to match the time for collecting data and the time in which the multiplication concept is taught. Thus the topic of multiplication, in this class, comes later in the syllabus than in other senior classes in which multiplication of whole numbers is learnt and taught. At each school, 8 learners were involved in semi-structured interviews about the representations they used in the tests they wrote. This resulted in interviewing 16 learners from the two schools. Three standard six Mathematics teachers were also involved.

3.6. Sampling Techniques

The schools were chosen because they are nearer to where I stay to minimize transport costs. Learners were randomly selected based on the representations they used; two of the learners who used no representations, two of the learners who used long multiplication, two of the learners who used repeated addition and two of the learners who used array representation in solving either of the problems in the pre-test.

Purposive sampling for selecting teachers was used; those who were involved were also teaching in standard 6 during the research period.

3.7. Pilot Study

A pilot study was conducted to 5 standard six mathematics teachers and 10 standard six learners of different schools. These participants were not taken from the schools that were involved in the main study. The teachers completed the questionnaires and the learners wrote the pre-test and were also interviewed after the test.

From the pilot study, it was discovered that teachers perceive that the use of multiple representation in multiplication of whole numbers help learners to understand the concept. Though these representations are perceived in this way, it has been noticed that multiple representations are not used when multiplying whole numbers in the senior section of primary education. From the pilot study, all the participants including the teachers used only one representation on the multiplication questions that were given to them. The representation that they used is the long multiplication. When asked why they used the long multiplication, the learners answered that, this is the only representation that their teachers used during multiplication of whole numbers. For questions, 1a: if 5x20=100, what is 5 x 18? and 1b: if 7 x 8=56, what is 7x16, no learner used the information (if 5 x 20 = 100 and if 7 x 8 = 56) that was provided. It was later discovered that the use of the representations is based on what representations are available in the learner's textbook and the teachers' guide. Teachers too, confirmed this by saying that, they use long multiplication because it is the only representation used in senior primary mathematics text books and teachers' guide.

3.8. Data Collecting Instruments

Data was collected using structured questionnaires, pre-test, lesson observation, interviews, and post-test. All these instruments can be found in the appendices from page 77. Firstly, questionnaires were filled by the teachers and the pre-test was administered to the learners who were interviewed to justify the representations they used. The pre-test was administered when learners had already learnt the multiplication concept with their teachers before the study was done. Secondly, a workshop was done by the researcher on teachers when multiplication of multi-digit numbers was presented using different representations. The result of the workshop was linked to the change in understanding of the learners that was discovered after the post test. Thirdly, teachers were asked to re-teach the multiplication concept using two representations of their choice and the lessons were observed. Lastly, the post-test was administered to the learners and later, they were interviewed. The test items on pre-test and posttest were not the same because the study did not focus on performance but it focused on understanding of the concept of multiplication through the representations used. Teachers were not interviewed because the study had focus on the understanding on learners.

In order to find out the perception of teachers on the use of multiple representations of multiplication concept and what mathematical representations are used in the classroom, questionnaires and a pre-test were administered to the class teachers and the learners respectively. The questionnaires included both open and close ended questions. They were chosen because of their ability to allow subjects to give information out of their own conscious without influence of others. Rich qualitative data is obtained as open ended questions allow the respondent to elaborate on their answer (Friedman & Rosenman, 1974).

Pre-test had multiplication problems of one digit number multiplied by one digit number, two digits number multiplied by one digit number, and four digits number multiplied by three digit number as put in appendix B . This range of problems was used to find out the representations that are used by learners when solving multiplication problems in different difficulty levels. The test items were taken from the work in the syllabus for classes from standard 2 to standard 6 as they learn multiplication of whole numbers with different number of digits according to their levels.

A workshop on the representation used in multiplication of whole numbers was patronised by all the teachers who were involved in the research. During the workshop the representations that were discussed include number line, rounding up and compensating, repeated addition, array representation, area model partitioning, and double multiplication. The teachers commented that these representations are indeed useful to learners because some of them are easy to understand. After the workshop, lessons were taught and observed. The lesson observation was there to see how the teachers have implemented what they have gained from the workshop. Teachers were given an opportunity to use at least two representations of their choice. A post-test was carried out in the two schools, to find out if the use of multiple representations has brought any change on learners' understanding. The learners were interviewed based on the representations they used when solving the problems in the post-test. The interview was used to see if there was going to be a difference in understanding of multiplication concept after learning the concept of multiplication using one representation and after learning the same concept using multiple representations and to probe their understanding. The interviews were looking for an explanation of the used representations in solving the problems. To check learners' understanding, learners need to justify their representations in the problems given (Nisbet et al., 2003). Below is a table which summarizes the instruments that were used and the questions they intended to answer.

Table 1: Research tools, source of information, type of data and research questions.

RESEARCH QUESTION	DATA TYPE	SOURCE	TOOLS
1. What are the perceptions of teachers on the use of representations?	➢ Perception of teachers on the use of multiple representations	➢ teachers	➢ questionnaire
2.What representations of multiplication do teachers use?	➢ Types of representation used in multiplication of whole numbers	➢ Teachers ➢ learners	➢ Questionnaire ➢ pre test
3. What knowledge of mathematical representations do teachers have?	➢ Teacher's knowledge on multiple representations.	➢ Teachers	➢ Questionnaire

4. How do teachers select representations of multiplication?	➢ Factors affecting the selection of representations	➢ Teachers	➢ Questionnaire
5. How do learners understand the multiplication concept from the multiple representations?	➢ Types of understanding of multiplication concept	➢ learners	➢ pre-test ➢ post-test ➢ interview

3.9. Data Analysis and Presentation

The findings were analysed based on the tools that were used to collect the data.

Questionnaires were analysed by first reading throughout all the comments made in response to the open ended questions that were asked in the questionnaire and grouped them into meaningful categories. The responses which were difficult to be categorised meaningfully and the questions that were not answered were put in a category 'other'.

The pre-test and the post-test were analysed by coding the representations that were used as 0: no representation, 1: long multiplication, 2: repeated representation. 3: array representation and 4: partitioning. The answers that were written by the learners in the tests were coded as follows; 0: omitted, 1: incorrect answer, 2: partially correct and 3: correct. Micro soft excel was used to enter the coded data. The data was presented using tables, pie charts and graphs.

The recorded interviews were transcribed and analyzed by using thematic data analysis. Thematic data analysis is a qualitative analytic method for identifying, analyzing and reporting patterns within data (Braun & Clarke, 2006, p. 79). The process started with reading and rereading the transcribed data to get familiarised with it. Codes and categories were generated to come up with the themes which were used to produce the report. The codes were interpreted to come up with the statements which are presented in chapter four.

Lessons were observed using a lesson guide; see appendix E to find out the representations that were used during the teaching and learning process. The codes for representations which have been outlined above were used.

3.10. Ethical Consideration

Permission was sought from the District Education Manager, the Primary Education Advisor and the Head teachers of the primary schools involved. After an authorisation from the head teachers, responsible teachers were asked if they were willing to participate in the study. When the teachers accepted, the standard six learners were told to be part of the study. Confidentiality was ensured to them for their right to privacy and anonymity. To ensure anonymity, schools were labeled 1 and 2, teachers were also labeled A, B and C and learners too, were labelled1 to 135 for school 1 and 1 to 78 for school 2. The subjects were allowed to discontinue their participation when they wished to do so as Bulmer, (2008) suggest.

3.11. Limitations of the study

There were four limitations of the study. The first is that, the study was planned to involve three schools but along the way, one school did not manage to continue with the study so it was allowed to withdraw and two schools continued up to the end of the study. The school which withdrew made the study to source information from three teachers instead of five teachers since the school that withdrew has two mathematics teachers in standard six. The study could have learnt more from five teachers than it has done from three teachers. However, the data collected from the three teachers that remained was sufficient to answer the research questions.

The second limitation is that the type of experimental design used in the study does not consider other factors that may affect the results apart from the treatment. The change in the results is attributed to the treatment used.

Another limitation is that data for the study was collected during the rainy season hence some learners were not present during the pre- test and pos-test. Their absence, made the study not to sample learners for interviews from the whole class, and the study might have missed some interesting responses.

Lastly, the study planned to observe lessons according to the planned work of the teachers in their schemes and records of work instead it was done during the arranged time because data was collected when the multiplication concept was already taught and learnt. This might have affected how the teachers taught the concept, nevertheless the study captured the types of representations teachers use when teaching multiplication of whole numbers as intended.

3.12. Chapter summary

In summary, the chapter has discussed the design of the study as being experimental where by quantitative and qualitative methods were used. The study was conducted at Boma educational zone of Dedza district which is found in the Central West Education Division. The study involved standard six primary school learners and standard six mathematics teachers. The study engaged a maximum number of 216 subjects which comprised of 213 learners and 3 teachers. Data was collected by using structured questionnaires for teachers, pre-test for learners, workshop for the teachers, lesson observation, and interviews with learners and post-test for students. Data was analysed and presented according to the tool that were used. Lastly, the chapter has discussed ethical consideration of the respondents and the limitations of the study.

CHAPTER FOUR

FINDINGS AND DISCUSSION

4.1. Introduction

This chapter discusses results obtained from the respondents. This discussion follows the order of the research questions of the study. In this view, the discussion starts with the demographic information of the respondents who were involved in the study before the discussion of the results.

4.2. Demographic Information

This section contains the sex and age range of the respondents.

a. Sex of Respondents

The study involved 113 males and 103 females.

Figure 1: Sex of Respondents

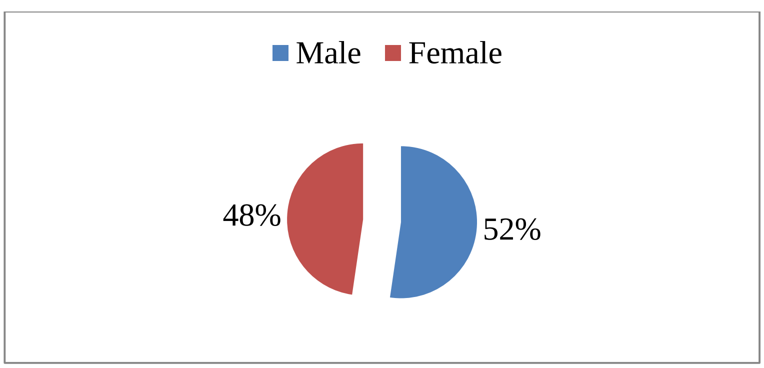


Figure 1 above shows that out of 216 respondents, 52% were males and 48% were females.

b. Age range of respondents who were learners.

Table 2: Age range of learners

Age group	**Frequency**
10 - 14	199
15 - 19	14

Table 1 above, shows that, most (93%) of the standard six learners were within the range of 10 and15 which is the modal class of the frequency table for the data.

4.3. Discussion of findings

This section discusses the findings of the study which were achieved according to the study questions. The main question for the study is; how are multiple representations used in teaching and learning of whole number multiplication? To answer this question, the following sub-questions were set:

1. What is the perception of teachers on the use of multiple representations?
2. What representations are used by teachers on multiplication of whole numbers?
3. What knowledge of multiple representations do teachers have?
4. How do teachers select representations of a concept?
5. How do learners understand the multiplication concept from the multiple representations?

To answer these questions, teachers' questionnaires, lessons that were taught and students' interviews, representations, performance on pre-test and post- test were analysed.

4.3.1. What is the perception of teachers on the use of multiple representations?

In order to find out the perception of teachers on the use of multiple representations, teachers completed written questionnaires. All teachers indicated that multiple representations are useful for learner's understanding of concepts. Teacher A from school 1 admitted that: *'The use of multiple representations is essential because some representations are helpful to the learners who have problems in memorizing the multiplication table. Other representations help those learners who are able to memorize the multiplication table.'* This indicates that learners use representations, they are able to use comfortably as Verschaffel (1994) say some learners favor visual or concrete representations, while others favor symbolic or abstract representations based on their ability.

Teacher B from school 1 said that: '*the use of multiple representations is very important because some learners understand what they are learning when they are able to see pictures of what is being taught. When a teacher draws objects like the array representation to stand for the numbers that are being multiplied, the learners have an opportunity to see that they can find the answer by either counting all the drawn objects or by adding the number of columns in number of row times or vice versa. This helps learners to understand that multiplication is repeated addition.'* This agrees with Bostrom and Lassen (2006) who say that many learners learn better when there are pictures to demonstrate how they can learn difficult and new knowledge.

Teacher C from school 2 said that: '*the use of multiple representations on multiplication of whole numbers is vital because there are many learners in primary schools who learn better when activities are involved in the lesson. When they learn the same concept with* *long multiplication, they remember and apply what they were doing in the activities.'* This supports the work of Bostrom and Lassen, (2006) who say that learners retain about 90% of what they say, as they do something in form of activities.

4.3.2. What representations are used by teachers on multiplication of whole numbers.

From the pre-test which was written by the learners, the questionnaires completed by teachers and interviews with learners, the study revealed that long multiplication, repeated addition and the array representation are the representations that are used on multiplication of whole numbers. The representations used by the learners are the ones that the teachers used in teaching multiplication in different classes. The learners revealed the representations used in particular classes during the interview. The following four problems were solved by the learners;

1a. if 5 x 20 =100, find 5 x 18,

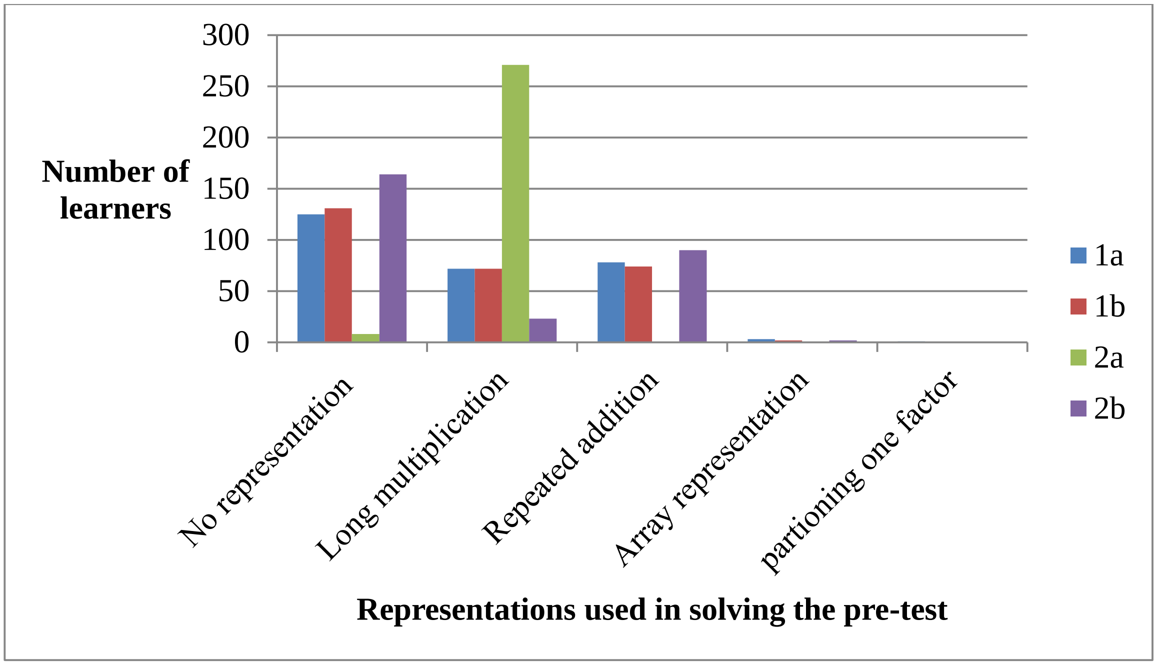
1b. if 7 x 8 = 56, find 7 x 16,

2a. represent 2712×149 in many ways as you can and

2b. represent 8×7 in many ways as you can

When solving these four problems, learners used the representations in figure 3 below:

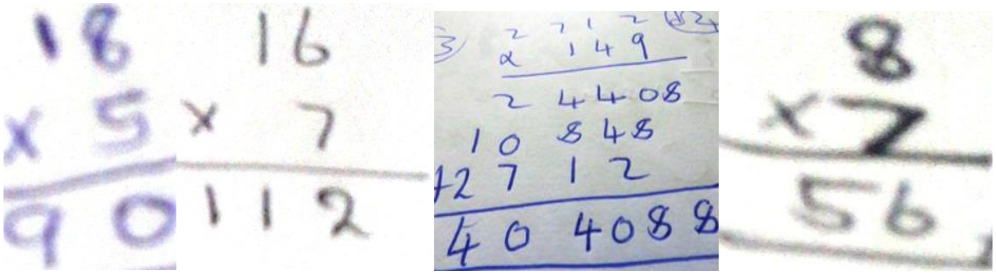
Figure 2: Representations used in multiplication of whole numbers.



4.3.2.1. Long multiplication

Since there were 4 problems that were solved by 213 learners, a total number of 852 answers were collected from different representations. Figure 3 shows that, most of the learners found their answers using the representation of long multiplication. Two hundred and eight learners representing 98% wrote the answers using this representation when solving problem 2a. Fifty learners representing 23% used this representation when solving problems 1b. Forty nine learners representing 23% used this representation for solving problem 1a and sixteen learners presenting 8% used this representation for solving problem 2b. Below are some examples of how the learners represented the four problems:

Figure 3: Long Multiplication representation for 1a, 1b, 2a and 2b.

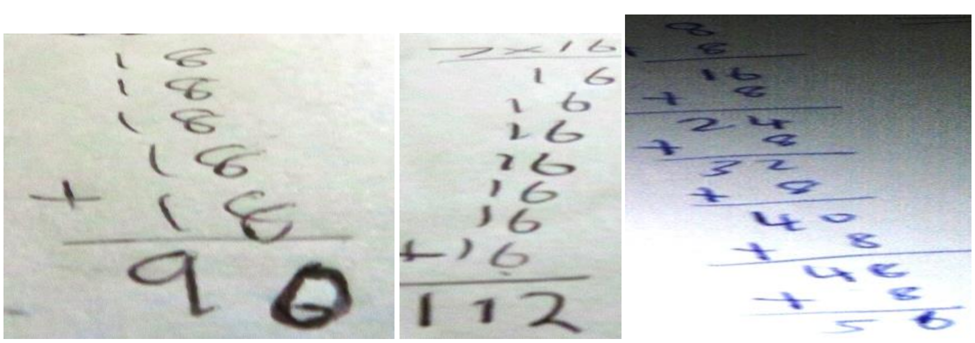


1a 1b 2a 2b

4.3.2.2. Repeated addition

It was also observed that, repeated addition was used by some of the learners to find answers of the problems. Ninety learners of about 42% used this representation when solving problem 2b. Seventy seven learners representing 36% got their answers for problem 1a by using this representation. Seventy three learners presenting 34% solved problem 1b using this representation. There was no learner who used this representation to solve problem 2a. The pictures below are showing some examples of how learners used repeated addition in the three problems.

Figure 4. Repeated addition representation for 1a, 1b and 2b

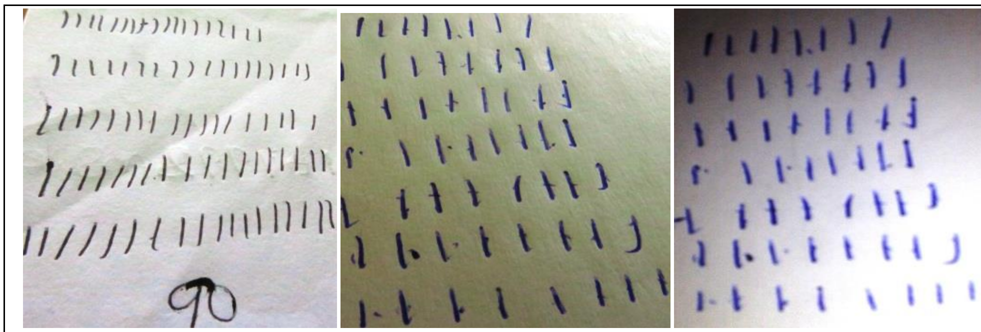


1a 1b 2b

4.3.2.3. Array representation

Another representation used by learners which the study has discovered is the array representation, though used by very few. Only two learners (1%) used this representation when solving problem 1a, 1b, and 2b. the two learners who used array representation in solving 1b and 2b were the same learners. No learner (0%) used the representation in solving problem 2a. These learners drew sticks to represent objects as shown in the following examples:

Figure 5. Array representation for 1a, 1b and 2b



1a 1b 2b

When explaining choice of the representations, the learners said that, they used these representations above in their learning of mathematics from standard 2 to 4.This has made the study to claim that in junior primary school where problems like 18×5, 16×7 and 8×7 are taught, multiple representations are used in the teaching and learning process. This agrees with Clements (1999) who says that different types of representations should be used concurrently in order to help the learning of learners for understanding. Wittmann (2013) also found out that, using enactive and iconic representations alongside symbolic representations one of which is long multiplication, is necessary not only for the so called slow learners. These representations are important for all learners and are useful throughout the entire learning process.

However it was discovered that all the learners including those who used the above representations did not use the information that was attached to the problems. As figure 3 indicates, 84 learners (39%) just wrote their answers with no representation for problem 1a, 88 learners (41%) did the same for problem 1b, 5 learners (2%) for problem 2a and 105 learners (49%) for problem 2b. when some of the learners were asked why they just gave the answers without using a representation, one learner said that *'ndinataimusila pa dela. Ndipo nditapeza ansala, ndinangotenga ansalayo ndikuilemba apapa. Ndinaona kut i chimene chikufunika ndi ansalayo'*, (I solved the problem on a separate worksheet. When I found the answer I just took the answer and wrote it here. I thought what was important was the answer only).

For standard 5 to 8, the senior section of primary school, the study revealed that learners learn multiplication of whole numbers through the use of long multiplication only. The interviews with the learners revealed that individual learners used long multiplication on problem 2a because it is the only representation they learnt on multiplying 4 digit number by 3 digit number. This supports the traditional didactics which states that, iconic and enactive representations are important in the early stages of learning, and as learners' age increases symbolic representation should take over. However, the view that all representation should be implemented at all stages is gaining more and more support (Clements, 1999).This entails that, in the schools that were involved in the study, representations are used on multiplication of whole numbers based on Bruner's work which says that, learners learn concepts through a particular representation according to their stage of development (Bruner, 1966). Using Bruner's idea, relating the stages of development for learners and the levels of primary education, a learner is supposed to attain the adolescent stage in standard five since adolescents are the ones who are supposed to learn multiplication concepts through symbolic representations one of which is long multiplication. In the 1960s, Bruner's idea seemed to work since learners were reaching their adolescent stage while pursuing primary education which is contrary to what is happening nowadays as table 1 indicates that most learners who were involved in the study were less than 15 years old. Therefore, multiple representations need to be used in the senior section.

4.3.3. What knowledge do teachers have on multiple representations of multiplication of whole numbers?

Teachers were also given some problems to solve in order to compare learner's representations and the teacher's representations, to find out the teachers' knowledge of different representations for each of the multiplication problems presented to them. In their responses to the questionnaires, teachers admitted that they use repeated addition, array representations, factored representation and long multiplication. When they were asked to solve some problems using different representations, their work was summarized as follows (school 1 had teachers A and B and school 2 had teacher C.

Table3. Representations used by teachers in the two schools

Problem	Representation	Frequency	%	School	Teacher
10a: 18 × 8	Long multiplication	3	100	1 and 2	A, B, and C
	Repeated addition	3	100	1 and 2	A, B, and C
	Array representation	2	67	1 and 2	A and C
	Factor method	2	67	1 and 2	A and C
10b: 8249 ×721	Long multiplication	3	100	1 and 2	A, B, and C
	Repeated addition	0	0	1 and 2	
	Array representation	0	0	1 and 2	
	Factor method	0	0	1 and 2	

Table 2 shows that long multiplication and repeated addition were used by all (3) teachers in their different schools when solving problem 10a. Teachers A, and C from schools 1 and 2 also used array representation and factor method when solving the problem. The table also shows that all teachers used long multiplication only when solving problem 10b. The pictures below show these representations.

Figure 6. Representations for 10a by teachers

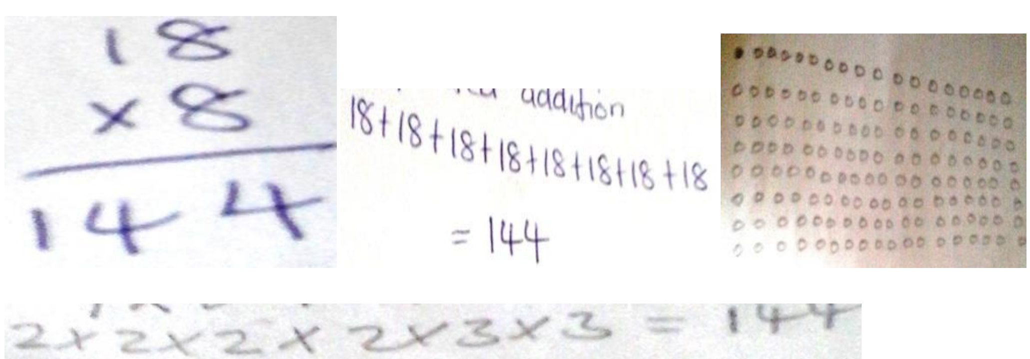


Figure 7. Representation for 10b by teachers

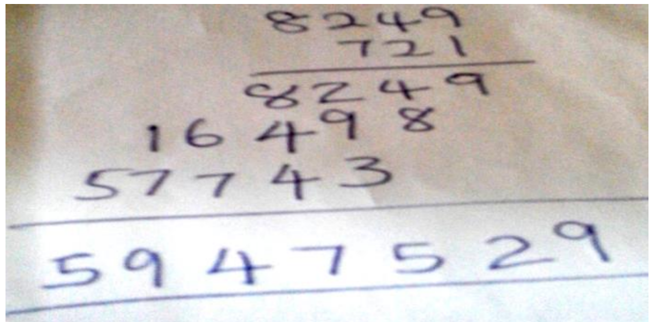


Table 2 above has helped the study to reveal that teachers of the two primary schools seem to lack the knowledge of multiple representations of multiplication of multi-digit numbers. Though the teachers show that they have knowledge of multiple representation on multiplication of a whole number by a single digit number, the knowledge does not seem to be broad enough. The reason might be their sources of representations are the teachers' guide and the learners' text books and these sources have insufficient number of representations. They only have the representations of array, repeated addition and long multiplication. Ball (1990) found that in order for a teacher to generate and use representations, he or she needs to have enough collection of representations that are useful for teaching mathematics, which would include stories, pictures, situations, and concrete materials. This shows that teachers have more common content knowledge than the specialised content knowledge.

4.3.4. How do teachers select representations of a concept?

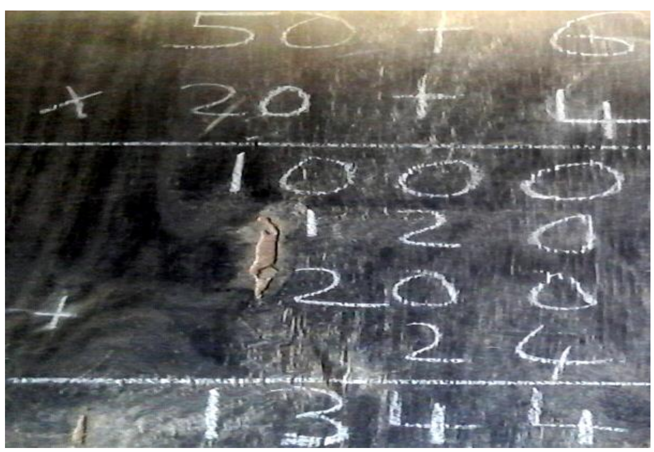
From teachers' responses on the question of the source of knowledge on representations, which was part of the questionnaire, all the teachers indicated that their source of knowledge is the teacher's guide and the text books. In addition, when the representations of multiplication of whole numbers from text books and teachers' guides for both junior and senior section were compared with the representations that teachers use in these sections, there was no difference, in other words teachers were just following what is in the teachers' guides and the text books. This shows that the teachers do not exercise the skill of selecting representations instead they just follow what the books are saying. This idea is supporting Debrenti (2013) who says that, teachers rely on course books most frequently when teaching accompanied by workbooks and study guides.

From the representations that were used by learners in the pre-test, the study revealed that teachers use representations based on the prior knowledge of learners on representations of a given concept. As it has been noticed that in the junior section long multiplication, array representation and repeated addition are used, the only representation that learners can use when multiplying multi-digit numbers as in problem 2a is the long multiplication. The reason behind this is that, it may be tiresome work for learners to add 2712 repeatedly 149 times or to add 149 repeatedly 2712 times. It can also be tiresome for learners to draw an array with 2712 columns by 149. It may be suggested that repeated addition or array representation are hard to be used in solving problems like the one in 2a (2712×149) of the pre-test. If other representations, like grid method, partitioning, and rounding and compensating, were introduced in the junior section, then the senior section could not have limited number of representations for representing multiplication of multi-digit numbers. This shows that while understanding different representations is important, some of the representations such as repeated addition and array representations are basic so it is expected that learners in standard 6 should know the connection between different representations and use most appropriate representation when given a whole number multiplication problem. For example, standard 6 learners are not expected to use repeated addition when multiplying large numbers but they can use partitioning or double multiplication. This is in line with Leinhardt's (1989) study which discovered that experienced teachers use representations that learners already know, to teach new concept. In this case the new concept is the multiplication of multi-digit numbers and the only possible representation that learners know is the long multiplication.

4.3.5. Lessons on multiplication of whole numbers

Teachers in school 1 introduced the lesson by testing the memory of learners on multiplication of single digit numbers. The main body was presented by using partitioning and grid method. The example that was used is 56 ×24. Using partitioning, it was represented as follows:

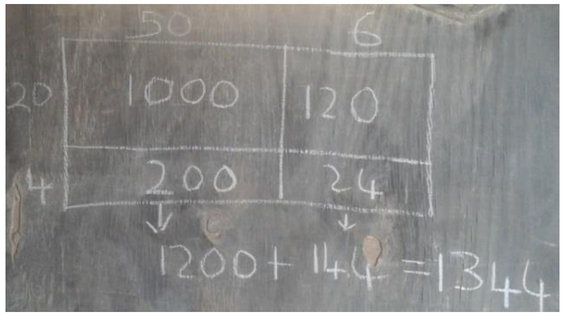
Figure 8. Partition representation



56 was partitioned to 50 + 6 and 24 was partitioned to 20+4 and then 50 was multiplied by 20 which gives 1000, 6 was multiplied by 20 to give 120, 50 was multiplied by 4 to give 200 and 6 was multiplied by 4 to give 24. At the end 100, 120, 200 and 24 were added together to get 1344. The teacher who did this emphasized much on place values when

partitioning the numbers by explaining that in 56 there are 5 tens (giving 50) and 6 ones while in 24 there are 2 tens (giving 20) and 4 ones. The same 56×24 was also done using grid representation with the partitioned numbers as follows:

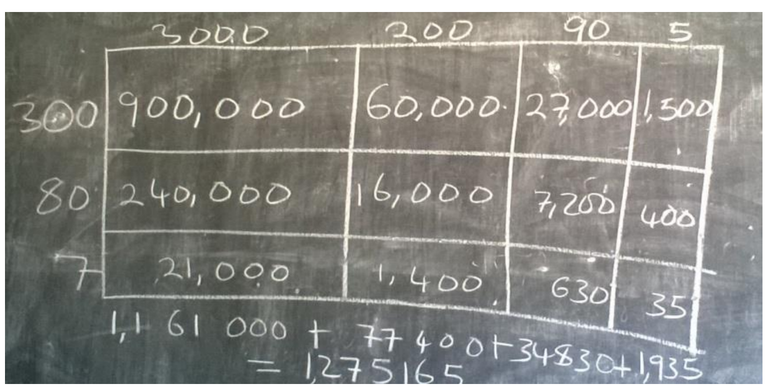
Figure 9. Grid representation with partition



From the partitioned numbers, the teacher solved the problem by first finding the partial products of the partitioned numbers. Fifty and six from 56 was written horizontally and twenty and four were written vertically. The products of 50 and 20, 6 and 20, 50 and 4 and 6 and 4 which are 1000, 120, 200 and 24 were added together to give 1344.

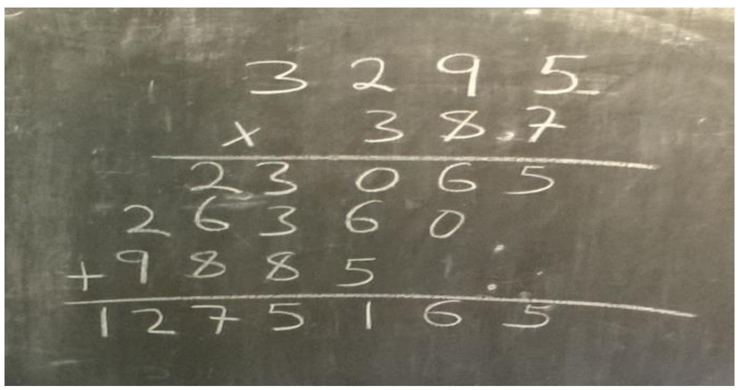
Teacher C from school 2 taught the concept of multiplication of whole numbers using grid method and long multiplication. This teacher used an example 3295×387 and it was represented as follows:

Figure 10. Grid representation used by teacher C.



This teacher C with the learners first partitioned the numbers 3295and 387 to 3000, 200, 90 and 5 and 300, 80 and 7 respectively. The partitioned numbers were presented in grids. Partial products were found by multiplying 3000 and 300, 200 by 300, 90 by 300, 5 by 300, 3000 by 80, 200 by 80, 90 by 80, 5 by 80, 3000 by 7, 200 by 7, 90 by 7 and 5 by 7. The partial products that were found are; 900000, 60000, 27000, 1500, 24000, 16000, 7200, 400, 21000, 1400, 630 and 35. After adding the partial products the answer 1275165 was found. The same teacher represented the same problem using long multiplication as in the picture below:

Figure 11. Long multiplication representation by teacher C



This teacher did not explain why the answer of 8 multiplied by 3295 slides over so that 0 is below 6 but learners followed the representation.

4.3.6. How do learners understand the multiplication concept from the multiple representations?

In order to find out how learners understand the multiplication concept from the multiple representations, the representations that were used by learners when solving problems in the pre-test and the post-test were compared and learners' responses for their interviews were compared too. The table below shows the representations used by the learners and their performance of the multiplication problems they were given.

Table 4a: Performance and representation by learners in the pre-test

School	Representation	5 x 18 ✓	5 x 18 ×	What is 7 x 16 ✓	What is 7 x 16 ×	2712 x 149 ✓	2712 x 149 ×	8 x 7 ✓	8 x 7 ×
1	No representation	73	8	49	33	0	5	77	14
	Long multiplication	18	0	16	1	62	68	9	0
	Repeated addition	31	3	30	4	0	0	29	1
	Array representation	2	0	2	0	0	0	2	0
	Grid method	0	0	0	0	0	0	0	0
	Partitioning	0	0	0	0	0	0	0	0
	Partitioning one factor	0	0	0	0	0	0	0	0
2	No representation	3	0	4	2	0	0	11	0
	Long multiplication	29	2	27	6	39	0	7	0
	Repeated addition	37	6	36	3	0	39	56	4
	Array representation	0	0	0	0	0	0	0	0
	Grid method	0	0	0	0	0	0	0	0
	Partitioning	0	0	0	0	0	0	0	0
	Partitioning one	0	0	0	0	0	0	0	0
	Factor	0	0	0	0	0	0	0	0

Table 4b: Performance and representation by learners in the post-test

	Representation	97 x 13 ✓	97 x 13 ×	137 x 53 ✓	137 x 53 ×
1	No representation	0	4	1	0
	Long multiplication	42	20	47	25
	Repeated addition	2	0	0	0
	Array representation	0	0	0	0
	Grid method	2	7	6	14
	Partitioning	0	0	0	0
	Partitioning one factor	24	31	16	26

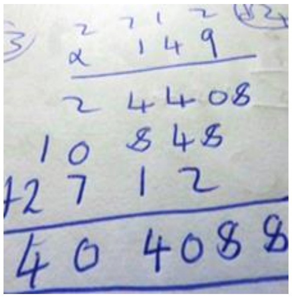
2	No representation	1	1	0	0
	Long multiplication	37	14	37	16
	Repeated addition	0	3	0	1
	Array representation	0	0	0	0
	Grid method	11	11	10	12
	Partitioning	0	0	0	0
	Partitioning one factor	0	0	1	1

Key

✓ Number of correct answer × Number of incorrect answers

From Table 3a, both schools solved the pre- test problems by using a number of representations except problem 2a which was solved by using one representation only. When the learners were asked to explain their representations, some learners showed that they do not know the reasons for carrying out some steps. Some learners' explanations for the long multiplication representation for problem 2a are shown below:

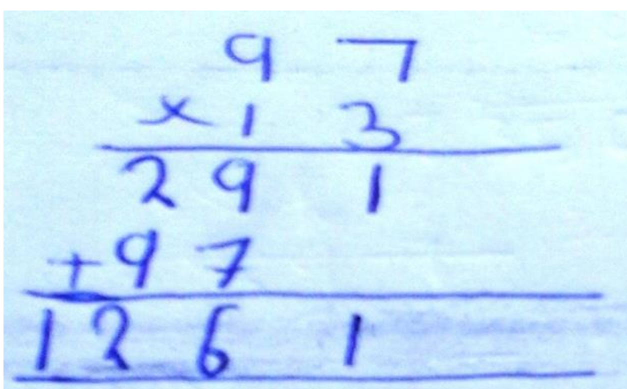
Figure 12. Long multiplication Representation for 2a



Some learners said when a product of two numbers in a multi-digit multiplication exceeds 9, the number which is on the right is written down and the one on the left is carried over. For example, in the problem above one learner said that "*potaimusa 2712 ndi 149, poyamba ndinapanga 9 taimusi 2 ndipo ndinapeza 18 ndiye ndinalemba 8* *ndikusunga 1 chifukwa 8 ali kumanja kwa 1",* (When multiplying 2712 by149 firstly, I multiplied 9 by 2 which gave me 18 so I wrote 8 and kept 1 because 8 is to the right of 1). The learners' explanations also showed that they do not know why the product of the other numbers in a multi-digit number shifts to the left. For example, another student said '*Kenako ndinataimusa 4 ndi 2 ndinapeza 8 ndipo ndinamulemba mmusi mwa 0 laini yachiwiri chifukwa choti 4 ali pachiwiri',* (then I multiplied 4 by 2 and got 8 so I wrote it under 0 in the second line because 4 is on second position). These explanations were similar to the explanations that were given on the long multiplication representation for problems 1a, 1b and 2b. As the explanation articulates, it seems that learners just know what to do when multiplying whole numbers by using long multiplication without understanding the reasons behind the procedures which is known as instrumental understanding. This agrees with Kemp (1976) who says instrumental understanding takes place when one uses an algorithm without really knowing how it works. As table 2 indicates, at least 48% of the learners who used long multiplication on all the 4 problems in the pre-test found the correct answer but with explanation that was not clear.

It was interesting to hear an explanation of the same long multiplication representation in the post-test which was as follows:

Figure 13. Long multiplication for a post-test problem



A learner from school 1 explained that *'potaimusa 97 ndi 13, ndinayamba kutaimusa 3 ndi 7 ndinapeza 21, ndiye mu 21 muli ma tens awiri, ndi ones mmodzi ndiye ndinalemba 1 amene ndi ones wathu, nkusunga ma tens awiri aja',* (when multiplying 97 by 13, I first multiplied 3 by 7 and I got 21 so in 21 we have 2 tens and 1 ones so I wrote 1 which is our ones and kept the 2 tens). The learner continued explaining that '*1 taimusi 7 ndi 7, chifukwa choti 1 ndi tens, 7 ndinamulemba mmusi mwa nambala ya tens',* (1 × 7 is 7 and since the 1 I have multiplied by 7 is on the tens value, I wrote the 7 under the tens number). After being asked how she knew this, the learner continued to explain that '*izizi ndazidziwa momwe timaphunzira kutaimusa nambala pogwiritsa ntchito njira ya partitioning, ndi pamene ndimakumbukira kuti paja pa nambala pamakhala ma ones, ma tens, ma hundreds nkumapitiliza eti'?,* (I have known this when we were learning the multiplication of numbers by partitioning, it is when I remembered that in a number we have ones, tens, hundreds and so on). From these explanations, the study suggests that presenting a concept using multiple representations transforms a learner from a state of understanding instrumentally to the state of understanding relationally by connecting skills that are done in one representation to another representation. In other words, multiple representations work as a connector of ideas within representations which facilitate learners' understanding. In relational understanding a learner knows how and why an algorithm works (Skemp, 1976). Understanding exists along a continuum, from an instrumental understanding (knowing something by rote or without meaning) to a relational understanding (knowing what to do and why) (Skemp, 1978). Instrumental understanding, at the left end of the continuum, shows that ideas are learned, but in isolation to other ideas, like multiplication was taught in isolation of the idea of place values. At this end there are ideas that have been memorized. Due to their isolation, poorly understood ideas are easily forgotten and are unlikely to be useful for constructing new ideas. At the right end of the continuum is relational understanding. Relational understanding means that each new concept or procedure is not only learned, but it is also connected to many existing ideas so there is a rich set of connections.

More interestingly in table 3b for the post-test, there was no learner from school 1 who used partitioning of the numbers given, though one of the representations that were used during the learning process was that of partitioning. Instead, 55 learners (41%) opted to partition one factor on problem 1a (solve 97 x 13) and 42 learners (31%) also partitioned one factor on problem 1b (137 x 53). Two learners (3%) from school 2 solved problem 1b by also partitioning one factor. Some learners partitioned the bigger factor while others partitioned the smaller factor, as the pictures below show how problems 1a and 1b in the post-test were solved by partitioning one factor.

Figure 14. Partitioning one factor for problem 1a: 97 x 13 in the post-test

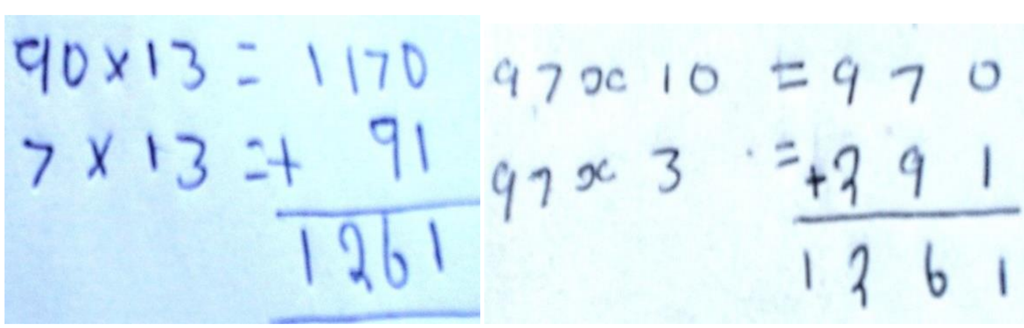
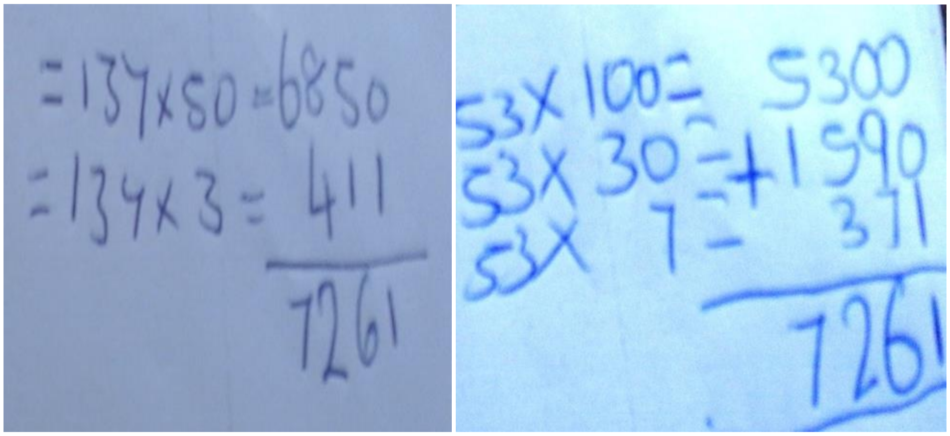


Figure 15.Partitioning one factor, for problem 1b: 137 x 53 in the post-test.

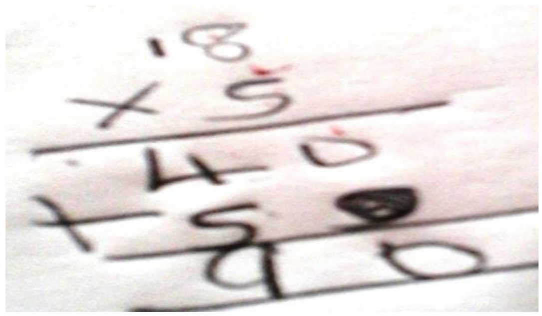


In partitioning of one factor, one factor is partitioned. From the example above, it is either 137 0r 53 which was partitioned. This representation reduces number of the partial products which are found. From this example there is 137 x 50 and 137 x 3. If 53 is partitioned to 50 and 3 or 53 x 100, 53 x 30 and 53 x 7 if 137 is partitioned to 100, 30 and 7. When the two factors are partitioned, number of partial products increases. When both 137 and 53 are partitioned, there might be 6 partial products which include; 100 x 50, 100 x 3, 30 x 50, 30 x 3, 7 x 50 and 7 x 3. It may be tedious to add the 6 partial products than to add the 3 or the 2 partial products.

For this representation, when one learner was asked to explain why he chose to partition one factor only, he said that (*ngakhale sindinaphunzireko kataimusidwe kanambala kugwiritsa ntchito njira imeneyi, ndinaona kuti nambala ikampwanyidwa simasintha tizidutswato tikaphatikizidwa ndiye kuphanya nambala imodzi kapena zonsei tipatsa ansala zofanana tikataimusa tizidutswato.* (Though I have not learned multiplication of whole numbers using this representation, I chose to use it because I saw to it that when a number is partitioned, it does not change its value when the partial numbers are multiplied, so partitioning one number or all numbers give the same answer after multiplying the partitioned numbers). When he was probed more to explain the exact reason which made him to use this representation not the partition of all the factors though they give the same answer, the learner continued that "*ndinaona kuti ndikopweka kutaimusa nambala ndinambala ina yoti yaphwanyidwa chifukwa timakhala ndi ma ansala ochepa oti tiphatikize"*. I found it simple to multiply a number with another number which has been partitioned because there is a small number of partial products which are supposed to be added. Another learner said that "*zimaphweka kutaimusa nambala ndi nambala ina yoti ikuthera zero monga ngati musamui 100, 50 ndi 30"*. (It is easier to multiply a number with another number which ends with zeros, in this example, 100, 50, and 30). Since It is easier to multiply a number with another number which ends with zeros, in this example, 100, 50, and 30) were able to use representation that was not used by the teacher, the study revealed that the multiple representations that were used in the learning process, helped the learners to understand the concept which made them to create their own representation. This supports the idea of Lamon (2001) who says that teachers may evaluate the learner's understanding of the concept if he or she uses a representation that has not been used by the teacher when teaching the concept. This entails that using multiple representation of multiplication of whole numbers help learners to become more creative and flexible when solving mathematical problems. This supports Piez and Voxman, (1997) view which says, learners should be given an opportunity to use representations they can invent or create. It is the reasoning and creativeness of the learners that made them realize that they can partition only one factor to make the calculation easier.

I was surprised to see one learner solving problem 1a: 18 x 5, of the pre-test using partitioning of the bigger number 18 as 10 + 8 as follows:

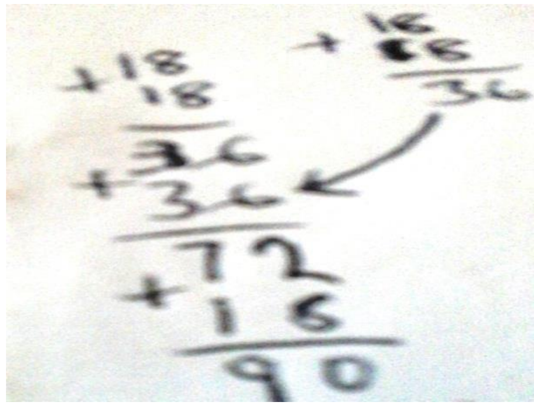
Figure 16. partition representation for 1a in a pre-test



When asked why he used that representation he said '*ndinangoganiza kuti ngati tinaphunzira njira zambiri zochulukitsira nambala ngati zimenezi, ndinaganizanso njira yanga kuti 18 ndi chimodzimodzi 10 + 8 ndiye kuchulukitsa ndi 5 tipeza 50 + 40'* (I just thought of it that if we learnt to multiply such numbers in different ways I also thought of my own way that 18 is the same as 10 + 8 so multiplied by 5 we get 50 + 40). In their study, Loveridge and Mills (2008) call those who use this representation as advanced multiplicative thinkers who will be able to use the distributive property of multiplication in future and can therefore construct and manipulate factors in response to a variety of contexts. The authors further say that such learners can derive answers to unknown problems from known facts, using the properties of multiplication.

For those who used repeated addition, there was a clear indication that the multiple representations, made the learners to think on how quickly they can come up with the answer. Zazks (2005) found that representations work as the tool for manipulation of numbers. Some students who used repeated addition on problem 1a represented it as follows:

Figure 17. Repeated addition for 1a in a pre-test



One of the learners who used this representation said that "*pa 18 x 5 pali ma 18 okwanira 5. ndikaphatikiza ma 18 awiri ndimapeza 36 ndiye kuti awiri enawonso andipatsa 36. Poti tachotsa ma 18 okwanira 4 ndiye kuti tatsala ndi18 mmodzi amene timuphatikizire ku zomwe ndapeza pophatikiza ma 18 okwanira 4 aja".* (Since there are five eighteens on 18 x 5, when I added two of them I got 36 and another two eighteens gave 36 and I was remaining with one eighteen which was added to the result of four eighteens). Note that the learner started with a representation (18+18+18+18+18) and deduced the conclusion based on another representation 36+36+18. This representation made the work of repeated addition to be easier because the second 36 can just be copied from the addition of the first eighteens and then being added together and the remaining eighteen. Thus this indicates the creativity that is being shown by the learners in simplifying some representations.

Since the study has found out that teachers cling to one type of representation, they do not link the multiplication of whole numbers with other topics like number bases and others when teaching, and for these reasons, the teachers have more CCK than SCK. From the pre- test, learners had no reasons for carrying out some steps when multiplying whole numbers and they were not able to create their own representations. This signifies that, the learners understood multiplication of whole numbers instrumentally when one representation was used. After being taught the same topic using several representations, learners were able to give reasons for a step, create their own representations and link long multiplication with number bases. This makes the study to conclude that, using multiple representations on multiplication helps learners to understand the concept relationally as the properties of relational understanding indicates on conceptual framework.

4.4. Chapter Summary

In summary, the study aimed at investigating the use of multiple representations in the teaching and learning of whole number multiplication. The results have confirmed that relational understanding is acquired in learners after learning the concept of multiplication of whole numbers using multiple representations. This means that, using multiple representations when teaching multiplication of whole numbers, yield positive results in understanding the multiplication concept. Using these representations, learners may connect skills and ideas among the representations, they may create their own representations and become flexible in using them when solving mathematical problems and they may use the representations as tools for operation of numbers.

CHAPTER FIVE

CONCLUSION

5.1. Introduction

This chapter concludes the thesis by giving a summary of the findings, discussing implications of the findings and suggesting areas for further study.

5.2. Summary of findings

The study has found out that teachers perceive that multiple representations are useful in the teaching and learning of whole number multiplication. This is because when teachers use multiple representations, learners choose to use a particular representation based on their ability; the choice is based on the representation that these learners may be able to use effectively. Another reason is that since some representations contain pictures, these pictures help some students to learn better when learning difficult concept. The other reason is that some students retain what they have learnt if activities are incorporated in the teaching and learning process since some representations contain activities.

For the question; what representations of multiplication do teachers use, the study has found that the representations that are used by teachers in the teaching and learning of whole number multiplication are: long multiplication, repeated addition and array representation. From these three representations, long multiplication was most commonly used and it was reported that the other representations are used in lower classes.

On the question; what knowledge of mathematical representations do teachers have, the study has found out that teachers who were involved in the study demonstrated limited knowledge of multiple representations of multiplication of multi-digit whole numbers. This was captured when the teachers were asked to multiply multi-digit whole numbers by using many representations that they know. The only representation they used is the long multiplication. In terms of Ball's framework, the teachers demonstrated more of Common Content Knowledge and little of Specialised Content Knowledge.

On how do teachers select representations of a concept, the study found out that, teachers do not practice how to select the representations, instead; they use whatever representations are in the teachers' guide and learners' text books. Since they had limited SCK, it is not surprising that they only followed what was in the text books and teacher's guide.

On how do students understand the multiplication concept from the multiple representations, the study has found out that using multiple representations in the teaching and learning of multiplication of whole numbers, has an important use. These representations change learners from being in a state of instrumental understanding to the state of relational understanding. Multiple representations help in this change of state by; connecting skills and ideas that are in one concept to another concept for example, learners used the ideas on place values on the concept of multiplication. Another way is that multiple representations play a role of helping learners to think and create their own representations from the ones used when learning and become flexible to use the representations for example, students were able to partition one of the factors being multiplied after learning multiplication through partitioning of all factors. Multiple representations also work as a tool for manipulation of numbers (Skemp, 1976) for example, students were able to come up with other numbers which were not in the problems they were asked to solve. Since the students demonstrated these, according to the framework, they had attained relational understanding (Skemp, 1976).

Lastly, the study has contributed towards the scholarly work by adding to the knowledge bank on the use of multiple representations since there is limited literature on this topic.

5.3. Implications of the findings

Based on the findings, the study has a number of implications. The first implication is that, using multiple representations in teaching and learning mathematical concepts may be interesting to all the learners. This is so because all learners may be captured in a lesson when multiple representations are used. Multiple representations engage learners of different ability since different representations capture interest of different learners, simplify a difficult mathematical concept for example learners choose representations which are easier to them as one learner said that it is easier to multiply a number with another number which ends with zeros that were found after partitioning the numbers and make learners to learn a new concept easily as they were able to learn multiplication of whole numbers using knowledge from place values. When learners are exposed to multiple representations, it is possible for them to develop relational understanding which can help them to apply the learnt concepts in other concepts. Therefore, it is important that multiple representations are used in teaching whole number multiplication. Furthermore, the teaching and learning of multiplication concept needs to be done in connection with other concepts like place values and number bases. This would make learners to be able to connect the concepts previously learnt to the one they are learning. As a result learner's interests may be captured and they may learn these concepts without difficulty.

The study has assumed that teachers are not fully exposed to the use of multiple representations in their teacher education which makes the teachers to use only the representations that are found in the textbooks and the teachers' guides. This results in failure to have another way of explaining the concepts to the learners who have problems in understanding the teacher's representation. Hence it is necessary that a number of representations should be accessible in teacher's guide and text books even for upper classes. This may help the teachers to present a concept in different ways. It is also important that teacher education should emphasise on the use of multiple representations when teaching and learning mathematical concepts. This may help the student teachers to emulate some of the representations that are presented by tutors. In line with this, teachers should be in a position to use multiple representations by involving them in continuing professional development, concerning the use of multiple representations. The representations may include; symbolic, verbal, pictorial or visual. Furthermore, teachers should be made to be able to create representations to improve the primary school learners' learning of mathematics. Such representations should comply with the various mathematical representations and focus on relational understanding. Lastly, the teachers should be encouraged to use other resources when planning their work, for example; they can use books from national libraries and internet on their phones.

5.4. Areas for further study

I suggest that further studies can also be done on multiple representations in the following areas:

a) An investigation of how teacher education can prepare teachers in the use of multiple representations in the teaching and learning of mathematics in primary school.

b) An investigation of the use of multiple representations in the teaching and learning of other topics in standard 6, to compare with findings from this study and to see if the findings are unique to multiplication of whole numbers.

5.5. Chapter Summary

This chapter has summarised findings of the study by briefly discussing the answers to the research questions. An acknowledgement of the study implications has also been discussed and finally some suggestions for further studies are given.

REFERENCES

Adu-Gyanfi, K. (1993). *External multiple representations in mathematics teaching*. USA: North Carolina state university.

Al-Asai, I. H. & Helai, A.A. (2015).The impact of using representations on acquisition of mathematical concepts among 6th graders in Gaza strip. *International journal of recent scientific research*, 6(3),3042-3049.

Ball, D. L. (2011*). What do math teachers need to know*? *Presentation for the Initiative For Applied Research in Education* . Retrieved from https://davidwees.com/conent

Ball, D. L. Thames, M. H. & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of teacher education*, 59(5), 389-407.

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449-4666.

Barmby, P., Bolden, D., Raine, S.,& Thompson, L. (2013). Developing the use of Visual representations in primary classroom. *Developing the use of Diagrammatic Representations in Primary Mathematics.* 55 (3), 13 – 32.

Bidwell, J. K. & Clason, R. G. (1970). *Readings in the history of mathematics education.* (403 – 407). Reston, VA: National Council of Teachers of Mathematics.

Bostrom, L. & Lassen, L. M. (2006). Unrevealing learning, learning styles, Learning strategies and meta-cognition. *In Education + Training*, 48(⅔), 178-189.

Box, G. E. P., Hunter, J. S. & Hunter, W. S. (2005). *Statistics for experimenters: Design, Innovation and Discovery.* London: Wiley-Interscience,

Braun, V. & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative research in Psychology*, 3(2), 77- 101.

Bruner, J. (1966). *Towards a theory of instruction* (Masters Thesis). Cambridge: Harvard University Press.

Bulmer, M. (2008). The ethics of social research, in Gilbert, N. (Ed.) *Research in social Life* (3rd ed.), London: Sage 145 -161.

Clark, J. M., & Paivio, A. (1991).Dual coding theory and education. *Educational Psychology Review, 3*(3), 149-210.

Clements, D. H. (1999).'Concrete' manipulatives, concrete ideas. *Contemporary Issues In Early Childhood*, *1*(1), 45-60.

Creswell, J. W. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches* (3rd ed.). Thousand Oaks, CA: Sage

Davis, B. (2008). Is 1 a prime number? developing teacher knowledge through concept study. *Mathematics Teaching in the Middle School*, 14(2), 86 -91.

Debrenti, E. (2013). Representations in primary mathematics teaching. *Actadidactica Napocensia*, 6, 55 – 64.

Devlin, K. (2000). *The math gene: How mathematical thinking is involved and why numbers are like gossip?* 48(2), 193 – 301.

Dienes, Z. P. (1971). *Building up mathematics*. London: Hutchinson Educational Ltd, p. 18-40.

Dlamini, E., Venkat, H. & Askew, M. (2015). A modeling and models approach: Improving primary mathematics learner performance on multiplication. *SAARMSTE*, (pp. 193 – 207). S.A.

Fennema, E., & Franke, M. L. (1992).Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (5th ed.). Reston, Virginia: National Council of Teachers of Mathematics.

Flevares, L. M. & Perry, M. (2001). How many do you see? The use of non spoken representations in first-grade mathematics lessons. *Journal of Educational Psychology*, 93(2), 330-345.

Friedman, M. & Rosenman, R. H. (1974). *Type a behavior and your heart*. New York: Knopf.

Goldin, G. A., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 1-23). Reston, VA: NCTM.

Janvier, C. E. A. (Ed). (1987). *Problems of representation in the teaching and learning of Mathematics.* Hillsdale, NJ: Lawrence Earlbaum Associates.

Kaput, J., J (1989).*Linking representation in the symbol systems of algebra.* Hillsdale, NJ: Earlbaum Associates.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Lamon, S. J. (2001). Presenting and representing: From fractions to rational numbers. In A. Cuoco (Ed), *The roles of Representation in School Mathematics*

(41- 52). Reston, NCTM.

Lasen, L. M., Bostrom, L. & Evans, C. (2006). *Young children use graphs to build mathematical reasoning.* Norway: University of Oslo.

Leinhardt, G. (1989). Math lessons: A contrast of novice and expert competence. *Journal for Research in Mathematics Education, 20*(1), 52-75.

Lesh, R., Landau, M., & Hamilton, E. (1983). Conceptual models in applied mathematical problem solving research. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (263-343). New York: Academic Press.

Loveridge, J. Y. & Mills, J. (2008).*Supporting multiplicative thinking. Multi- digit multiplication using array – based materials*. New Zealand: University of Waikato,

Marzano, R. J. (2004). Building background knowledge for academic achievement: *Research on what works in schools*. Alexandria, VA: Association for Supervision and Curriculum Development.

Marzano, R. J., Pickering, D. J., & Pollock, J. E. (2001).Classroom instruction that works*: Research based strategies for increasing student achievement*. Alexandria, VA: Association for Supervision and Curriculum Development.

Ministry of Education Science and Technology. (2005). *Primary school syllabus for Mathematics.* Domasi, Malawi Institute of Education.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Nisbet, S., Jones, G,. Thornton, C., Langrall, C. & Mooney, E. (2003).Children's representation and organisation of data. *Mathematics Education Research*, 15(1), pp. 42- 58.

Orton, R. E. (1988). Using representations to conceptualize teachers' knowledge. In M. J. Behr,C. B. Lacampagne & M. M. Wheeler (Eds.), *Proceedings of the Tenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 23-30). DeKalb, Illinois.

Ozgun-Koca, S, A. (1998).Student's use of representations in mathematics education. Raleigh: *Proceedings of the Psychology of Mathematics Education.*

Paulos, J. A. (1988). *Innumeracy: Mathematical illiteracy and its consequences.* Retrieved from https//jet.com/product/innumeracy/mathematics.

Piez, C. M. & Voxman, M. H. (1997). Multiple representations using Different perspectives to form a clearer picture. *Mathematics teacher*, 90(2), 164- 167.

Salkind, G. M. (2007). *Mathematical representations*.USA: George Mason University.

Sisakht, Z. G. & Larki, F. N. (n.d). *The role of a multi-representational instructions in the understanding of fractions in elementary mathematics.* Retrieved from https://www.researchgate.net/publication/265805851_the_role_of_a_multi-representational_instruction_in_the_understanding_of_fractions_in_elementary_mathematics/download

Skemp, R. R. (1979). Goals of learning and qualities of understanding. *Mathematics Teaching*, 88, 44 – 49.

Skemp, R. R. (1978). Relational understanding and instrumental understanding.

Arithmetic Teacher, 26, 9 – 15.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Research, 15*(2), 4-14.

Soon, G. K. (n.d). Development of mathematical concepts and skills in primary school children. *Teaching and Learning*, 3(1), 14-20.

Stake , S. R. (1995). *The art of case study research.* London: Sage publication.

Sunyono, Yuanita, L. &Ibrahim, M. (2015).Supporting students in learning with multiple representations to improve student mental models on atomic structure concepts. *Science Education International*, 26 (2) 104-125.

Verchaffel, L. (1994). Using retelling data to study elementary school children's representations and solutions of compare problems. *Research in Mathematics education*, 25(2), 141- 165.

Wallace, A. H. & Gurganus, S. P. (2005). Teaching for mastery of multiplication. *Teaching Children Mathematics*, 12, 26-33.

Wittmann, E. (2013). Standard representations in the teaching of arithmetic. *Mathematic-Didaktik*, 19(2), 23- 149.

Yerushalmy, M. (1997). Designing representations: reasoning about functions of two variables. *Research in Mathematics Education*. 28(4), 431-466.

Zazks, R. (2005). Representing numbers: prime and irrational. *International Journal* of *Mathematical Education Science and Technology*, 36, 207-218.

Appendix A: Questionnaire for Teachers

1. What are your views towards the use of multiple representation of multiplication of whole numbers?

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2. What representations of multiplication of whole numbers do you know?

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3. What is the source of your knowledge on representations?

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4. What representations do you use on multiplication of whole numbers from the ones mentioned in 2 above?

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5. Give reasons for your selection

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6. What challenges do you meet when using multiple representations?

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7. How do you think the challenges can be solved?

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8. What factors do you consider when choosing representations for a concept?

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9. Do you think using multiple representations on multiplication of whole numbers has an impact on student's understanding?

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   If yes how? And if no give a reason

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10. Represent the following in many ways as you can

    a. 8x18

    b. 8249x721

11. From the representations in 10 above, what representations can best be used in the problems?

12. Give reasons for your choice

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Thank you for your participation

Appendix B: Pre-test for Students

1. If 5x20 = 100, what is 5x18?

2. If 7x8 = 56, what is 7x16?

3. Solve the following using different representations.

   a. 2712 x 149?

   b. 8x7

Appendix C: Post- test for Students

Name ……………………………………………………

Solve the following:

a. 97×13

b. 137×53

Appendix D: Interview Questions for Students

1. How did you find your answer
2. Is there any other way that you know can be used to find the same answer
3. Why did you choose to use that way?
4. Why do you write the number found to the right hand side and keep the number that is to the left when the product exceeds 9?
5. What happens to a whole number when multiplied by another whole number apart from 1?
6. What ways of representations of multiplication of whole numbers have you learnt in :
    a. this class?
    b. Previous class?
7. Do the ways in 3 above help you to understand the multiplication of whole numbers?

    If yes how/ if no why?

Appendix E: Lesson Observation Guide

Introduction: Number of representations being used

Development: Are the representations helping the students to understand the concept of multiplication?

Are the used representations giving the same concept meaning?

Conclusion: Are the students able to use representations when solving Problems involving multiplication?

Appendix F: Interview Transcript for Teacher 1

Repeated is good to those who are unable to memorise the multiplication table. Long multiplication is good to those learners who are able to memorise the multiplication tables. Representations of multiplication of whole numbers that I know are long multiplication and repeated addition. I have known the multiplication from college, continuous professional development and teachers guide. I frequently use long multiplication when teaching whole number multiplication. I choose this representation because it is the one which is done very fast. The challenges that I meet when using this representation is that learners do not write their answers in the way they are supposed to be e.g. 711 instead of 711

711	711
x 23	x 23
2133	2133
1422	1422
3555	16353

This challenge can be solved by giving the students more practice e. g. homework and knockout. The factor that I consider when choosing representations for a concept is ability of the students. Yes, using multiple representation help students to understand the multiplication of whole numbers. I think in this way because if learners are taught different methods/ways of working out the problem, they are able to choose the method they feel is easier to them. Representations for 8 x 18 are:

i. Repeated addition

   18 + 18 + 18 + 18 + 18 + 18 + 18 + 18 = 144

ii. Long multiplication

$$\begin{array}{r} 18 \\ \underline{\times\ 8} \\ 144 \end{array}$$

For 8249 x 721 I have only long multiplication that I can use.

$$\begin{array}{r} 8249 \\ \underline{\times\ 721} \\ 8249 \\ 16498 \\ \underline{57743} \\ 5947529 \end{array}$$

For 8 x 18 the best representation is repeated addition and for 8249 x 721 is long multiplication. Repeated is better for 8 x 18 because it is not time consuming. Long multiplication is better for 8249 x 721 because it encourages a learner to be mentally active in solving problems by recalling multiplication table.

Appendix G: Interview Transcript for a Student After Pre-test.

For the question if 5 x 20 = 100, what is 5 x 18? I added 5 eighteens.

Ndinaphatikiza ma 18 okwanira 5 monga 18 + 18 + 18 + 18 + 18 .Nditaphatikiza ma 18 ndinapeza 90. Njira ina yotaimusira 5 ndi 18 ndikuyambira kaye 5 x 8 ansala ndi 40 ndiye timalemba 0 kusunga 4 poti 0 ndi amene ali kumanja kwathu. Kenako timataimusa 5 ndi 1 ansala ndi 5 pulasi 4 ansala ndi 9.

$$\begin{array}{r} 18 \\ \underline{\times\ 5} \\ 90 \end{array}$$

Ndinasankha njira yophatikizayo chifukwa ndi imene imandipwekera. Potaimusa 7 ndi 16, ndinaphatikiza ma 16 okwanira 7 monga 16 +16 + 16 + 16 + 16 + 16 + 16.Ndikaphatikiza ma 16wa ndimapeza 112. Njira ina yotaimusira 7 ndi 16 ndi kutaimusa kaye 7 ndi 6 ndipo ansala ndi 42 ndiye tikuyenera kulemba 2 ndikusunga 4. Kenako 7 taimusi 1 imatipatsa 7 pulasi 4 ikukwanira 11 ndiye ansala ndi 112. Ndinasankha njira yophatikiza chifukwa ndi imene inandiphwekera.

Potaimusa 2712 ndi 149, ndinayambira kutaimusa 9 ndi ma nambala onsewo, kenako kutaimusa 4 ndinambala zonse ndikumalizira ndikutaimusa 1 ndinambala zonse motere:

$$\begin{array}{l} 2712 \\ \underline{\text{X } 149} \\ 24408 \\ 10848 \\ \underline{2712} \\ 404088 \end{array}$$

Ndinagwiritsa ntchito njira imenei chifukwa ndi njira yokhayo imene taphunzira potaimusa nambala zikuluzikulu. Potaimusa 8 ndi 7 ndinajambula tindodo 8 m'mizere 7. Kuwerenga tindodo tonse timakwanira 56.

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Njira ina yotaimusira 8 ndi 7 ndikuphatikiza ma 8 okwanira 7 monga 8 + 8 + 8 + 8 + 8 + 8 + 8 amene akutipatsa 56. Ndinagwiritsa ntchito njira yojambula tindodo chifukwa choti 8 ndi 7 ndinambala zing'onozing'ono.

Timati tikataimusa nambala ndikupeza 10 kapena kupitililapo, timalemba nambala imene ili kumanja ndikusunga imene ili kumanzere chifukwa ndi mmene anatiphunnzitsila aphunzitsi athu. Tikataimusa ma nambala ansala imakhala nambala yaikulu kuposa nambala zomwe tikuzitaimusazo. Njira yotaimusira nambala imene ndaphunzira mkalasi muno ndi ija ndagwiritsa ntchito potaimusa 2712 ndi 149. Njira zomwe ndaphunzira mmakalasi am'mbuyomu ndi monga yophatikiza, yojambula tindodo ndi njira yotsitsa. Njira zinazi tinaziphunzira kuyambira standard 2, 3 ndi 4. Njira imene ndaphunzira mkalasi muno imandivuta kumva mwina chifukwa chakuti ndi yaitali komanso imafuna zinthu zambiri monga kusunthira kumanzere ukamataimusa ndi nambala yachiwiri. (when multiplying 5 x 18, I added 18s' which were 5 like 18 + 18 + 18 + 18 + 18. When I added the answer was 90. Another way of multiplying 5 and 18 is to start with multiplying 5 by 8 which gives 40 and 0 is written keeping 4. Zero is written because it is the one which is found to the right side. I chose to use repeated addition because it is the one which is simple to me.

When multiplying 7 by 16, I added 16s' which were seven in number like 16 +16 + 16 + 16 + 16 + 16 + 16 and I found 112. Another way of multiplying 7 by 16 is by first multiplying 7 by 6 which give 42 and 2 is written, 4 is kept because 2 is the one which is to the right of the other. Then, 7 is multiplied by 1 which gives 7, plus the kept four we have 11 so the answer is 112. I chose the repeated addition because it is the one which is simple to me. For 2712 x 149, I first multiplied 9 by each digit on 2712, and then I multiplied 4 by 2712 and then 1 by 2712.

  2712

X 149

  24408

10848

2712

404088

I used this representation because it only the one I know that can be used to multiply big numbers.

When multiplying 8 by 7 I drew 8 sticks in 7 lines and counted the number of sticks. There were 56 sticks. 8 and 7 may be multiplied by adding eights which are 7 in number.

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I chose this representation because the numbers that are involved are small numbers.

When multiplying numbers, if the answer is 10 or more, we write the number which is to the right and keep the one to the left because our teacher told us to do so. When multiplying whole numbers together, the answer is more than the factors which are multiplying themselves.

In this class I have learnt the representation that I have used when solving 2712 x 149. Previously in other classes I learnt repeated addition, drawing sticks and the one which goes down. We learnt these other representations in standards 2, 3 and 4. I am having difficulties with the representation I have learnt in this class because it needs a lot of things like shifting to the left when multiplying a multi-digit number by a second or the third digit of the other factor).

Appendix H: Interview Transcript for a Student After Post-test.

Potaimusa 97 ndi 13, ndinayambira 3 ones taimusi 7 ones ndipo ansala ndi 21 ones ndipo mu 21 muli ma tens awiri ndi ones mmodzi. Ndinalemba 1 ndikusunga 2. Kenako ndinataimusa 3 ndi 9 tens ndikupeza 27 tens pulasi ma tens awiri tinasunga aja tili ndi ma tens 29. Kenako ndinataimusa 1 tens ndi 7 ones ikutipatsa 7 tens ndipo ndinamulemba mmusi mwa 9. Kenako ndinataimusa 1 tens ndi 9 tens ndikupeza 9 hundreds. Kenako ndinaphatikiza nambalazo.

$$\begin{array}{r} 97 \\ \underline{\text{X } 13} \\ 291 \\ \underline{+\ 97} \\ 1261 \end{array}$$

Tikhoza kutaimusanso 97 ndi 13 pophatikiza ma 97 okwanira 13 monga 97 + 97 + 97 + 97 + 97 + 97 + 97 + 97 +97 + 97 + 97 + 97 + 97. Tikhonzanso kutaimusa 97 ndi 13 pophwanya 97 kukhala 90 ndi 7 komanso 13 kukhala 10 ndi 3. Kenako nambalazi tiziyika mzigawo za rectangle.

	90	7
10	900	70
3	270	21
	1170 +	91 = 1261

Ndinagwiritsa ntchito njira yoyambayo chifukwa ndi imene ndinaizolowera. Potaimusa 137 ndi 53 ndinapanga 137 x 50 + 137 x 3 imene ndinapeza 6850 + 411 imene ikukwanitsa 7261. Tikhozanso kutaimusa 137 x 53 popanga 100 x 50+ 30 x50 + 7 x 50 + 100 x 3 + 30 x 3 + 7 x3 imene ingatipatse 5000 +1500 + 350 + 300 + 90 +21 imene ikutipatsa 7261.

Ndinagwiritsa ntchito njira iyiyi ngakhale sitinaiphunzire chifukwa ndi njira ya ifupi. Tikamataimusa nambala ndikupeza 10 kapena kupitilirapo timalemba nambala yakumanja chifukwa tikapeza ansalayo timagawa ndi 10 ndiye timalemba ma rimenda.Ndakumbukira izizi pamene timaphunzira njira yotaimusira nambala pogwiritsa ntchito ma place value. Njira zomwe ndaphunzirazi zanditsegula maso kuti samu ikhoza kusovedwa ndi njira zosiyanasiyana ifeyo ngati ana a sukulu tikonza kumaganizanjira zathu ngakhale sizinaphunzitsidwe ndi aphunzitsi. Komanso njirazi zandiphunzitsa kuti samu zataimusi ndi za place value zimagwirizana. (when multiplying 97 by 13, I started with multiplying 3 ones by 7 ones and the answer was 21 ones and in 21 we have 2 tens and 1 ones. I have written 1 and kept 2 tens. Then I multiplied 3 by 9 tens and found 27 tens adding to the 2 tens that I kept I have 29 tens. Then I multiplied 1 tens by 7 ones and I got 7 tens which I wrote below 9. Then I multiplied 1 tens by 9 tens and found 9 hundreds and I added the numbers.

$$\begin{array}{r} 97 \\ \underline{\times\ 13} \\ 291 \\ \underline{+\ 97} \\ 1261 \end{array}$$

We can also multiply 97 by 13 by adding 97s which are 13 in number like 97 + 97 + 97 + 97 + 97 + 97 + 97 + 97 +97 + 97 + 97 + 97 + 97. We can also multiply 97 by 13 by partitioning 97 to 90 and 7, and 13 into 10 and 3. The partitioned numbers can be put in grids.

	90	7
10	900	70
3	270	21

1170 + 91 = 1261

I used the first representation because it is the one I am used to. When multiplying 137 by 53 I multiplied 137 by 50 + 137 by 3 which gives 6850 + 411 which give 7261. We can also multiply 137 by 53 by 100 x 50 + 30 x 50 + 7 x 50 + 100 x 3 + 30 x 3 + 7 x 3 which may give us 5000 +1500 + 350 + 300 + 90 +21which may result to 7261.i chose the representation of partitioning only 53 though we did not learn this representation because it is short. When multiplying number and find 10 or more we write the number which is to the right because we divide the number by 10 so we write the remainder. I remembered this when we were learning multiplication by applying the knowledge of place values. The representations that I have learnt have opened my eyes to see that a problem can be solved by different representations and as students we should think of our own representations though not taught by the teacher. I have also learnt from these representations that multiplication is connected to the knowledge of place values.